Lessons 1 -2

Suggested Answers

Suggested Answers

Pre-lesson Worksheet

1. No specific answer and usually the ratio is approximately 1:1

2. All the ratios are approximately 1.41:1

3. (a) Height =11.4 inch, width =15.2 inch (b) Height = 9.31 inch, width =16.66 inch
4. (a) 3:2 (or 1.5:1) (b) 0.67

Task 1 (Set 1) Flower petals

Name	Number of petals
White Calla Lily	**1**
Euphorbia	**2**
Trillium	**3**
Wild rose	**5**
Clematis	**8**
Black-eyed Susan	**13**
Plantain	**21**

Task 1 (Set 2) Honeybees

In a colony of honeybees there is a special female called the **queen**.

There are many **worker** bees who are female too but unlike the queen bee, they produce no eggs.

There are some **drone** bees who are male and do not work.

Males are produced by the queen's unfertilized eggs, so male bees only have a mother but no father!

All the females are produced when the queen has mated with a male and thus have two parents.

So female bees have 2 parents, a male and a female; whereas male bees have just one parent, a female.

Number of	parents	Grand-parents	Great grand-parents	Gt, gt grand-parents	Gt,gt,gt-grand-parents	Gt, gt,gt, gt-grand-parents
Male bees	**1**	**2**	**3**	**5**	**8**	**13**
Female bees	**2**	**3**	**5**	**8**	**13**	**21**

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Suggested Answers

Task 2

Knowledge Review: $T_1, T_2, T_3, T_4, \cdots, T_n, \cdots$ are the terms of sequence.

1. List the number of parents (including grand-parents and etc) in male honeybees as a sequence.
   1, 2, 3, 5, 8, 13, 21

2. Predict the next two terms of the above sequence. **34, 55**

   Hence, find the recursive formula of the sequence.

$T_1 + T_2 = 1 + 2 = 3 = T_3$

$T_2 + T_3 = 2 + 3 = 5 = T_4$

$T_3 + T_4 = 3 + 5 = 8 = T_5$

$T_4 + T_5 = 8 + 13 = 21 = T_6$ i.e. $T_{n-1} + T_{n-2} = T_n$

3. What's the type of the sequence? **Fibonacci Sequence**

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Task 1 (Set 3) Rabbits

Suppose a newly-born pair of rabbits, one male, one female, are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits. Suppose that our rabbits **never die** and that the female **always** produces one new pair (one male, one female) **every month** from the second month onwards.

1. At the end of the first month, how many pair(s) will there be? **1**
2. At the end of the second month, how many pair(s) will there be? **1**
3. At the end of the third month, how many pair(s) will there be? **2**
4. At the end of the fourth month, how many pair(s) will there be? **3**
5. At the end of the fifth month, how many pair(s) will there be? **5**
6. How many pairs will there be in one year? **144**

1,1,2,3,5,8,13,21,34,55,89,144

Task 2

Knowledge Review: $T_1, T_2, T_3, T_4, \cdots, T_n, \cdots$ are the terms of sequence.

4. List the number of petals (Task 1) as a sequence.

1, 2, 3, 5, 8, 13, 21

5. From the above sequence,

$T_1 + T_2 = 1 + 2 = 3 = T_3$

$T_2 + T_3 = 2 + 3 = 5 = T_4$

$T_3 + T_4 = 3 + 5 = 8 = T_5$

$T_4 + T_5 = 8 + 13 = 21 = T_6$

Using the above patterns, $T_{n-1} + T_{n-2} = T_n$

This sequence is called **Fibonacci Sequence**.

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Suggested Answers

Task 3

1. Using the above Fibonacci sequence to give your answers correct to 4 significant figures if necessary,

$$\frac{T_2}{T_1} = 2$$

$$\frac{T_3}{T_2} = 1.5$$

$$\frac{T_4}{T_3} = 1.667$$

$$\frac{T_5}{T_4} = 1.6$$

$$\frac{T_6}{T_5} = 1.625$$

$$\frac{T_{11}}{T_{10}} = 1.618$$

$$\frac{T_{50}}{T_{49}} = \frac{12586269025}{7778742049} \approx 1.618$$

2. If we continue in this pattern, the ratio will approach the decimal number __**1.618**__.

   This value is approximate to the golden ratio.

Golden Rectangle

Golden Rectangle is a rectangle that can be cut into a square and a rectangle **similar to** the original one.

Figure 1 shows that *ABCD* is a ***Golden Rectangle*** such that *FCDE* is similar to *ABCD*.

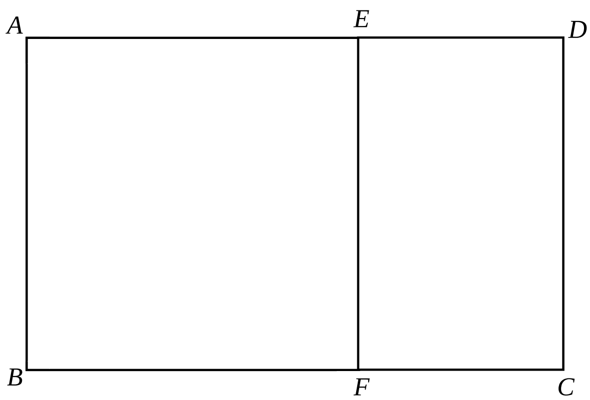
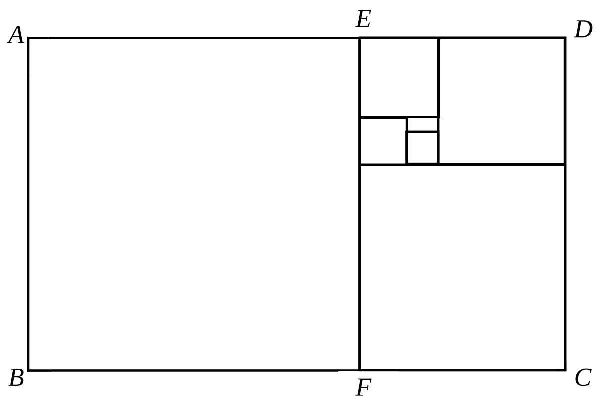


Figure 1



Similar rectangles: ratios of corresponding lengths equal

Golden rectangle: the ratio of length to width is equal to golden ratio

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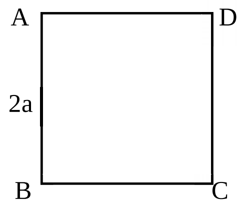
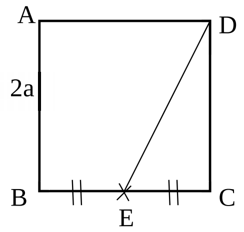
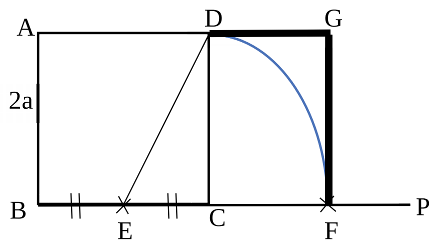
Suggested Answers

<u>Task 4 (Set 1) Exact value of Golden Ratio</u>

Form Golden Rectangles from a square ABCD with side 2a.



Step 1	**Step 2**	**Step 3**
→E is the mid-point of BC →Join ED	(i) Extend BC to point P (ii) Use a compass to draw an arc from D (where E is the centre) to line CP, mark a “x” in the point of intersection and call this point F	(i) Draw a line which is parallel to CD and passes through F (ii) Extend AD to the above line until they meet at G
		

(1) Express ED, BF and CF in terms of a. $ED = \sqrt{5}a$, $BF = (\sqrt{5}+1)a$, $CF = (\sqrt{5}-1)a$

(2) Are $\frac{BF}{AB}$ and $\frac{FG}{CF}$ equal? Explain briefly.

$$\frac{BF}{AB} = \frac{\left(\sqrt{5}+1\right)a}{2a} = \frac{\sqrt{5}+1}{2}$$

$$\frac{FG}{CF} = \frac{2a}{\left(\sqrt{5}-1\right)a} = \frac{2}{\sqrt{5}-1}\cdot\frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{\sqrt{5}+1}{2}$$

Yes! This value is the exact value of golden ratio.

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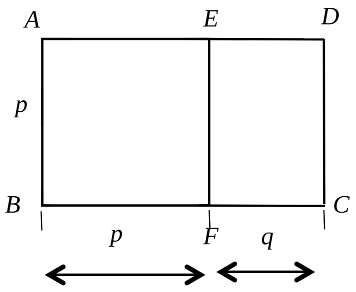
Suggested Answers

Task 4 (Set 2 and Set 3) Exact value of Golden Ratio

The figure shows that *ABCD* is a ***Golden Rectangle*** such that *FCDE* is similar to *ABCD*.



Using $\dfrac{BC}{AB}=\dfrac{CD}{FC}$ and let φ be $\dfrac{p}{q}$

(i.e. $\dfrac{1}{\varphi}=\dfrac{q}{p}$), **form** and **solve** an equation in φ.

$$\frac{BC}{AB}=\frac{CD}{FC}$$

$$\frac{p+q}{p}=\frac{p}{q}$$

$$1+\frac{q}{p}=\frac{p}{q} \rightarrow 1+\frac{1}{\varphi}=\varphi$$

$$\varphi^2-\varphi-1=0$$

$$\varphi=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-1)}}{2}$$

$\varphi=\dfrac{1\pm\sqrt{5}}{2}$ because φ must positive

i.e $\varphi=\dfrac{1+\sqrt{5}}{2}$

Useful information

For the equation $a\varphi^2+b\varphi+c=0$, $\varphi=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Example

Solve $2\varphi^2-5\varphi+3=0$.

$$2\varphi^2-5\varphi+3=0$$

$$\varphi=\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(3)}}{2(2)}$$

$$=\frac{5\pm\sqrt{25-24}}{4}$$

$$=\frac{5\pm\sqrt{1}}{4}$$

$\varphi=\dfrac{5+1}{4}$ or $\varphi=\dfrac{5-1}{4}$

$\varphi=\dfrac{3}{2}$ or $\varphi=1$

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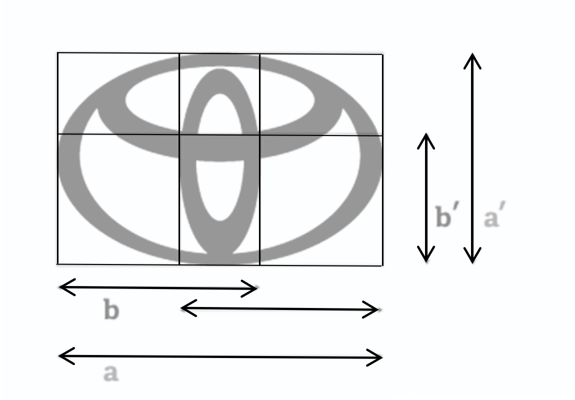
Suggested Answers

Task 5 Golden Ratio in real life

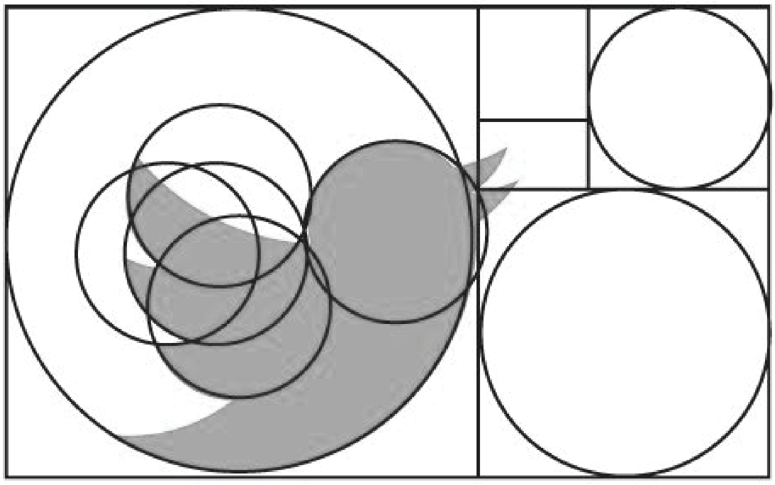
(I) Ratio in Design



By measurement,

$$\frac{a}{b}=\frac{5.1}{3.2}\approx 1.59$$

$$\frac{a'}{b'}=\frac{3.3}{2.1}\approx 1.57$$



Let d_l, d_m and d_s be the diameters of the largest circle, middle-sized circle and the smallest circle respectively.

$$\frac{d_l}{d_m}=\frac{7.4}{4.5}\approx 1.644$$

$$\frac{d_m}{d_s}=\frac{4.6}{2.9}\approx 1.5862$$

Could the logo have been designed with the golden ratio?

Yes, the ratios are close to the golden ratio

(II) Ratio in Architecture

1. The Parthenon in Athens.

$$\frac{BC}{AB}=\frac{9.8}{6.1}\approx 1.6066$$

$$\frac{CD}{FC}=\frac{6.1}{3.8}\approx 1.6053$$

$$\frac{DE}{GD}=\frac{3.8}{2.3}\approx 1.6522$$

2. Some people claim that it is based on golden rectangles. What do you think?

   Yes, some ratios are close to golden ratio