Golden Ratio and Fibonacci Sequence

Mathematics Secondary 2

Level 1: School-based Whole-class Teaching

Background and Notes

The design of the learning and teaching plan reflects the actual circumstances of the particular school at the time of implementation. As it is developed and tailor-made to meet the specific cognitive and affective needs of students, all learning and teaching resources are for reference only.

When adapting the materials, curriculum, instructional and assessment modifications can be made in accordance with the diverse needs and abilities, learning styles and aspirations of students, professional competence of teachers, and gifted education development of the schools.

Teachers are strongly recommended to read the introduction, theoretical background and summary of the resource package to have a better understanding of the principles of Gifted Education and strategies for implementation.

This unit includes 1 foreword, 1 lesson plan, 5 sets of worksheets, 1 set of suggested answers and 1 set of hints.

With reference to our resources, educators can design suitable learning activities and implement the elements of Gifted Education, based on students' needs and interests, and teaching experience, so as to unfold students' potentials to the fullest.

Golden Ratio and Fibonacci Sequence

Grade: Secondary 2
No. of Lessons (Learning Time): 2 Consecutive Lessons (80 minutes in total)

Prior Knowledge	- Basic knowledge of ratio - Pythagoras' Theorem - Fibonacci Sequence - Similar Figures - Surds
Learning Objectives	- Students can investigate the relationship between Golden Ratio and Fibonacci sequence - Students explore the application of Golden Ratio in real life - The aesthetic sense of the students can be developed
Intended Learning Outcomes	- Students deal with the counting problem in a systematical way - Students discover that the ratio of successive terms approaches to a value - Students are willing to challenge themselves - Students are willing to present their findings - Students appreciate one another during group discussions and presentation
Learning & Teaching Strategies	Ability Grouping, Presentation
Operation Mode of Gifted Education	Level 1: School-based Whole-class Teaching
Core Elements of Gifted Education	Higher-order Thinking Skills Creativity Personal-social Competence

Foreword / Background

The Project School has classes of about 30 students with mixed Mathematical ability. Most students have a good foundation in Mathematics and are willing to engage with challenging Mathematics lessons. In each class, there are about 5 to 10 students with outstanding performance in Mathematics or gifted in Mathematics. These students have regular training in a pull-out Mathematics programme in the school. Seeing the potential of the students, teachers would like to enrich and extend the regular curriculum so students can find the lessons more challenging and engaging. Their idea matches Level 1B of the three-tier implementation model for gifted education in Hong Kong.

When students were in S1, teachers gave a rough introduction to the 'Fibonacci sequence' when they were teaching the topic 'Sequence'. As students are learning the topic 'Ratio' in S2, teachers would like to introduce the Golden Ratio as an enrichment topic. The Golden Ratio is closely related to the Fibonacci Sequence. To study the relation, students need to apply knowledge across different Mathematics topics like Pythagoras' Theorem, ratio and similar figures. To study the exact value of the Golden Ratio, advanced topics like quadratic equations and irrational numbers are also needed. Education Bureau (2017) suggested that the aims of the Mathematics Education KLA Curriculum include helping students develop number sense, measurement sense, the capacity to appreciate structures and patterns and an appreciation of the aesthetic nature of Mathematics. The study of the Golden Ratio and the Fibonacci Sequence involves recognition of number patterns and appreciation of the aesthetic aspects of Mathematics. Hence, the Golden Ratio and the Fibonacci Sequence can be a suitable topic for providing enrichment and challenge for these students.

Objectives of Collaboration

The aim of the collaboration is to design a lesson for the whole class with the topic Golden Ratio and Fibonacci Sequence. The lesson aims to expose students to advanced Mathematics topics and challenging problems. As students like to take up a challenge, the lesson would be designed to have more student-centred problem-solving tasks rather than direct teaching. To solve the problems, students are expected to apply knowledge across different Mathematics topics. Through these problems, they are also expected to refine their problem solving and Mathematical thinking skills.

Students with high Mathematical ability or students gifted in Mathematics can have a chance to unleash their potential when solving more challenging problems. When advanced topics are introduced, these students can also extend their learning. During the problem-solving activities, these students might also have a chance to take up leading roles such as guiding other classmates or presenting their solution to the whole class

Theoretical Framework

The above objectives cannot be easily reached in a whole-class setting. Teachers are encouraged

to adopt differentiated instruction strategies. Differentiated instruction means that teachers proactively plan and carry out varied approaches to the learning content, process and product in response to student differences in readiness, interest, and learning needs (Tomlinson, 2005).

Tomlinson (2005) has developed a graphic tool called an "Equalizer" which comprises nine continuums along which the difficulty level of lesson content, process, or product may be located. It can help teachers expand the repertoire of ways they think about varying the challenge level of a specific task. When teachers design tiered learning tasks or resources to respond to differences in student readiness, they are trying to adjust the difficulty level of the task so that all students experience a challenge that is neither too great nor too small. The gifted and high ability students can experience a more challenging task where their potential can be unleashed. Teachers, with reference to students' ability and characteristics, can adjust the curriculum or teaching strategies by moving the control to the most suitable position to meet the different learning needs of students.

Tomlinson (2005) also suggested that differentiated instruction is a blend of whole-class, group, and individual instruction. Teachers monitor the match between learner and learning activities and make adjustments to guarantee effective matches.

Learning and Teaching Strategies

In the lesson, students were grouped according to their Mathematical ability and tiered assignments were prepared to provide an appropriate fit for different ability groups. Following the suggestion by Education Bureau (2017) for preparing a tiered assignment, teachers may first consider the instructional level of average students. Then the assignment can be modified to become more challenging by increasing the level of difficulty and complexity for the Mathematically gifted students. Thus, the lesson was first designed to have the following learning tasks:

Task 1	Study scenarios about Fibonacci Sequence in nature
Task 2	State with formal Mathematics notation the definition of Fibonacci Sequence
Task 3	Find the ratios of consecutive terms in Fibonacci Sequence and determine whether the value approaches a certain value
Task 4	Understand the definition of Golden Rectangle and Golden Ratio and find the exact value (in surd form) of Golden Ratio
Task 5	Study scenarios about Golden Ratio in design and architecture

These five learning tasks were then developed into three sets of worksheets to match the ability groups. In the three sets of worksheets, Task 2, Task 3 and Task 5 are almost the same while Task 1 and Task 4 are different. Task 1 and Task 4 in different sets of worksheets were designed to have a varied challenge level based on the Equalizer.

For Task 1, the teacher developed 3 sets of tasks using three different scenarios about the Fibonacci Sequence in nature. For the groups with lower ability, they study a scenario about flowers by counting the pedals from the picture. It is more concrete and simple. For medium and high ability groups, they study scenarios which require students to list and count things systematically. The tasks for these two groups are more complex and abstract.

Task 4 is the most challenging part of the lesson, the groups with lower ability are given a task requiring them to do a geometric construction using rulers and a pair of compasses. Then, they need to apply Pythagoras' Theorem and knowledge about surds to find out the exact value of the Golden Ratio. For the medium ability and high ability groups, they need to make use of quadratic formula to find out the exact value. The task for lower ability groups requires smaller leaps while that for the other two groups requires greater leaps. Also, a more concrete example about quadratic formula and more hints are provided in the worksheet for medium ability groups while nearly no hints are given to the high ability groups. Hence, the high ability groups are more independent compared to the medium ability groups. Furthermore, hint cards are prepared to better support students in need and extra learning materials (online video clip, extension worksheet) are prepared for groups that finish earlier.

Task 5 is not a difficult task but much time is needed for students to measure the lengths with rulers. Hence, the teacher planned to introduce the task in the lesson and leave it with the students as a post-lesson exercise.

As some of the tasks are different among the groups, the time for student presentation will be reserved so that students know what other groups are studying and learn from one another. Students with good presentation skills can be assigned to be presenters prior to the lesson. Hence, the lesson would involve both small group discussion, whole class instruction and student presentation.

Discussion

The lesson was conducted in two classes in the Project School. The Project GIFT team participated in one of the classes as observers. Students finished Task 1 to Task 3 smoothly. Only a few groups needed hints from teachers. Some faster groups were provided with I-Pads to view extra learning materials while waiting for the others. For Task 4, the most challenging task, only a few groups could finish without teacher's help. The teacher provided hint cards to the groups at different times depending on students' progress. For some groups, the teacher sat in and guided the students to solve the problem. After the presentation for Task 4, time was up and Task 5 was left as a post-lesson exercise. Overall, all students were highly engaged in the learning tasks and their

presentation of findings were impressive. The teacher maintained a facilitative role and wrapped up different parts of the lesson skilfully. The results of lesson observation indicated that the designed curriculum content and learning activities were effective in enhancing students' engagement in solving challenging problems.

The teacher who conducted the lesson reflected that it was hard to estimate the time required by students for the learning tasks. That was why she prepared some I-Pad and online learning materials for students. It was also difficult to support all the groups in a class with 30 students. Teachers who would like to conduct similar lesson are advised to make good use of hint cards. Teachers can also allow students to decide whether they need hint cards. To facilitate group works, the teacher pre-assigned group leaders and presenters so students did not waste time assigning roles in the lesson. Observers found this measure very useful.

To conclude, if students have good foundations and interest in Mathematics, teachers can enrich the curriculum so students can learn beyond the regular curriculum. With good adjustment of the learning tasks, all students, not only gifted students or students with high ability, can benefit from the provision of such enrichment.