Unit A

Suggested Answers and Guidelines

Suggested Answers and Guidelines for Unit A

Pre-lesson Worksheet 1

Task 2

Proof Since (a, b, c) be a Pythagorean triple,

we have $a^2 + b^2 = c^2$.

Hence,

$$a^2 = c^2 - b^2$$
$$= (c+b)(c-b)$$

So both $c+b$ and $c-b$ are factors of a^2 .

But a is a prime, so the only factors of a^2 are 1 , a and a^2 .

Hence there are only two cases:

Case 1: $c+b = a$ and $c-b = a$

Then : $c+b = c-b$

$b = 0$ and $c = 0$, which is not possible.

Case 2: $c+b = a^2$ and $c-b = 1$

So we must be in case 2, and the second equation gives $c = b+1$. *(end of proof)*

Question: Is the converse of the **Proposition** true?

Let (a, b, c) be a Pythagorean triple , where $a < b < c$ and $a^2 + b^2 = c^2$.

If $c = b + 1$, is it true that a is always a prime number?

[Counter-example: (9 , 40 , 41)]

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Lesson Worksheet 1

Observe the following triples and answer the questions:

Primitive Pythagorean triple (a, b, c)

m	n	(a) m^2-n^2	(b) $2mn$	(c) m^2+n^2
2	1	3	4	5
3	2	5	12	13
4	1	15	8	17
4	3	7	24	25
5	2	21	20	29
5	4	9	40	41
6	1	35	12	37
6	5	11	60	61
7	2	45	28	53
7	4	33	56	65
7	6	13	84	85
8	1	63	16	65
8	3	55	48	73
8	5	39	80	89
8	7	15	112	113
9	2	77	36	85
9	4	65	72	97
9	8	17	144	145
10	1	99	20	101
10	3	91	60	109
10	7	51	140	149
10	9	19	180	181
11	2	117	44	125
11	4	105	88	137
11	6	85	132	157
11	8	57	176	185
11	10	21	220	221
12	1	143	24	145
12	5	119	120	169
12	7	95	168	193
12	11	23	264	265
13	2	165	52	173
13	4	153	104	185
13	6	133	156	205
13	8	105	208	233
13	10	69	260	269

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Task 1 **Pattern of** $\dfrac{(c-a)(c-b)}{2}$

For a primitive Pythagorean triple (a , b , c) , what kind of number $\dfrac{(c-a)(c-b)}{2}$ should be?

Q1) Choose some sets of primitive Pythagorean triples, evaluate $\dfrac{(c-a)(c-b)}{2}$.

1, 4, 9, 16, …

Q2a) What kind of numbers the $\dfrac{(c-a)(c-b)}{2}$ are?

Findings: $\dfrac{(c-a)(c-b)}{2}$ must be a square number.

Q2b) Can you prove your suggestion? [Hint: use the formula $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$

Proof:

$$\begin{aligned}\frac{(c-a)(c-b)}{2} &= \frac{(m^2+n^2-m^2+n^2)(m^2+n^2-2mn)}{2}\\ &= \frac{2n^2(m^2-2mn+n^2)}{2}\\ &= n^2(m^2-2mn+n^2)\\ &= n^2(m-n)^2\\ &= [n(m-n)]^2\end{aligned}$$

Q3) Is the converse true? If not, give an example.

Converse:

If $\dfrac{(c-a)(c-b)}{2}$ is a square number , then (a , b , c) is a primitive Pythagorean triple.

Not true

e.g. $\dfrac{(3-1)(3-2)}{2} = 1$ is a square number while (1, 2, 3) is not a Pythagorean triple.

To construct such example, teacher can guide students to start with a square number,

e.g. : 4

$$\frac{(c-a)(c-b)}{2} = 4 \quad \Rightarrow (c-a)(c-b) = 8$$

Look for factors of 8 , $8 = 1\times 8$ and 2×4 , then make a $(c-a)(c-b)$ pattern.

e.g. $2 \times 4 = (7 - 5) (7 - 3)$, while (3, 5, 7) are not a Pythagorean triple.

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$\boxed{\text{Task 2}}$ ***a* , *b* and *c* , odd or even?**

Properties of Pythagorean Triples

Q1) Can all a , b and c be odd?

No. As "odd" + "odd" = "even"

Q2) As the primitive Pythagorean triple are coprime, can all a , b and c be even?

No. They should be coprime

Q3) Try to summarize how a , b and c be odd or even.

Exactly one of a, b is odd; c is odd.

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$\boxed{\text{Task 3}}$ **Height of Triangle with length (a , b , c).**

Given a primitive Pythagorean triple (a , b , c) , if we construct a triangle with length a , b and c .

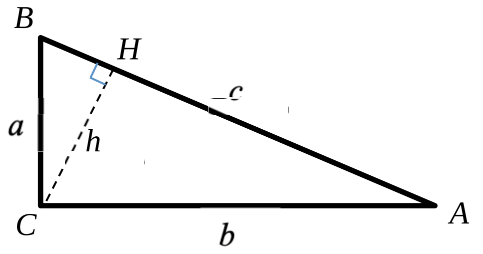
Q1) What kind of triangle is it?

Right-angled triangle.

Q2) Let h be the altitude (the height) of the triangle with respect to c .

Express h in terms of a , b and c .

Method 1 . Area relation



$$\frac{ab}{2}=\frac{ch}{2}$$

$$ab=ch$$

$$h=\frac{ab}{c}$$

Method 2 . Similar Triangle

$\Delta ACH \sim \Delta ABC$

$$\frac{CH}{BC}=\frac{AC}{AB}$$

$$\frac{h}{a}=\frac{b}{c}$$

$$h=\frac{ab}{c}$$

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$\boxed{\text{Task 4}}$ **Factors of a , b and c**

Q1) Can you observe any factors of a , b and c ?

- Exactly one of a, b is divisible by 3.
- Exactly one of a, b is divisible by 4.
- Exactly one of a, b, c is divisible by 5.

Q2) What is the largest number that always divides abc ?

60

Q3) Observe c . What is the general pattern of c ?

- c is in the form $4n + 1$.

Extra Materials:

1. Following Task 4, there is a reason why the base 60 system is used in the history.
   You can watch the video.

   Why We Still Use Babylonian Mathematics and the Base 60 System

2. Pythagorean triple can be studied in the view of complex number.
   You can watch the following two videos to understand more.

   Introduction to complex numbers

   Finding Pythagorean triples from square of Complex numbers