賽馬會「知優致優」計劃
Jockey Club "Giftedness Into Flourishing Talents" Project

Extension of Pythagoras' Theorem

Mathematics Secondary 2 or 3

Level 2: School-based Pull-out Programme

CENTRE FOR UNIVERSITY & SCHOOL PARTNERSHIP
大學與學校夥伴協作中心

香港賽馬會慈善信託基金
The Hong Kong Jockey Club Charities Trust
同心同步同進 RIDING HIGH TOGETHER

Background and Notes

The design of the learning and teaching plan reflects the actual circumstances of the particular school at the time of implementation. As it is developed and tailor-made to meet the specific cognitive and affective needs of students, all learning and teaching resources are for reference only.

When adapting the materials, curriculum, instructional and assessment modifications can be made in accordance with the diverse needs and abilities, learning styles and aspirations of students, professional competence of teachers, and gifted education development of the schools.

Teachers are strongly recommended to read the introduction, theoretical background and summary of the resource package to have a better understanding of the principles of Gifted Education and strategies for implementation.

This unit includes 1 foreword, 1 lesson plan, 8 worksheets, 3 sets of suggested answers and 1 set of extra materials.

With reference to our resources, educators can design suitable learning activities and implement the elements of Gifted Education, based on students' needs and interests, and teaching experience, so as to unfold students' potentials to the fullest.

Extension of Pythagoras' Theorem

Grade: Secondary 2 or 3
No. of Lessons (Learning Time): 3 Lessons (60 -90 minutes for each unit)

Operation Mode of Gifted Education	Level 2: School-based Pull-out Programme
Target Students	- S2 or S3 Students having outstanding or above-average performance and strong interest in Mathematics - Students who are gifted in Mathematics - High-potential students with great with high initiatives in learning Mathematics and good foundations in Mathematics and reasoning skills, as observed by teachers

Foreword / Background

Through daily observation, some students of the Project School have been found to have strong interest in mathematics. They are keen on answering questions and sharing their ideas in the lesson. They are also enthusiastic about attempting challenging questions and may even continue the discussion about questions at recess and lunchtime. However, without the extended training that suits these students, their learning progress may be limited to textbook content, the examination syllabus and the regular lessons. They need systematic training that brings insight into the subject and sustains their learning interest. This coincides with the rationale of the Level 2 implementation of school-based gifted education, which is to conduct a pull-out programme to nurture talents of students (Education Bureau, n.d.).

Pythagoras' Theorem is a topic covered in the S2 mathematics curriculum. It is an important and well-known theorem in geometry. Some areas of mathematics such as number theory and trigonometry have close relations with the theorem, so it can be a starting point to widen students' exposure to and understanding of mathematics, and to nurture their thinking and generic skills.

Objectives of Collaboration

Learning and teaching in regular classrooms are usually restricted by pre-defined learning content and limited lesson time. In contrast, a pull-out programme can have more flexibility to incorporate learning activities that train students to think like a mathematician. This pull-out programme aims to focus on process skills and to promote self-learning in mathematics. The learning units in this programme highlight different inquiry and problem solving skills in mathematics, such as

generalization, observing patterns and making judgements by proof or counter-example. These skills are essentials in learning advanced mathematics and can pave roads for further self-learning. After the lesson, students are expected to develop a mindset that learning of mathematics does not end at calculation, but is extended to exploration or review of knowledge from different perspectives.

Student Selection Criteria and Procedures

The pull-out programme targeted on S2 or S3 students having outstanding or above-average performance and strong interest in mathematics, with proper selection criteria and procedures.

1. Mathematics examination results

The programme coordinator first listed some targeted students according to previous mathematics examination results to select the high achievers and above-average students.

2. Teachers' nomination

Based on the list, mathematics teachers of each class commented on students' learning attitude and their interest in mathematics. Teachers also suggested other students with strong interest in mathematics but not listed. The following shows some possible characteristics of students gifted in Mathematics according to EdB (n.d.). Teachers can refer to these characteristics when looking for target students. They:

- Learn faster than ordinary students
- Demonstrate high levels of comprehension power in mathematics
- Are interested in figures and signs
- Enjoy abstract thinking
- Manipulate mathematical concepts easily
- Feel bored to learn by means of memorization or drilling
- Are able to find shortcuts to solve problems
- Are able to adopt flexible and multiple approaches to solve mathematical problems

3. Students' self-nomination

Students who are interested could also make self-nomination.

Theoretical Framework

According to VanTassel-Baska and Stambaugh (2006), a gifted programme must be designed to offer deep content learning. It should include a major emphasis on process skills such as problem solving, critical thinking and research skills. When designing a curriculum for the gifted, educators can apply the following features of differentiation to the content areas, namely acceleration, complexity, depth, challenge, creativity, and abstractness.

Features	Application to Content Areas
Acceleration	The pacing of the programme can be adjusted to decrease the speed of learning and increase the depth, or to increase the speed by requiring fewer tasks to master a standard, which allows the student to pursue advanced content.
Complexity	Additional variables, multiple resources or more difficult questions may be posed. Students are required to practise higher-order thinking skills.
Depth	Students are required to apply concepts in multiple ways and generate knowledge by themselves.
Challenge	The content discussed can be more sophisticated and require a larger amount of reasoning.
Creativity	Students can be asked to construct a model based on a concept studied, have opportunities to complete alternative tasks or products of their choosing, or represent new learning in their personal choice of mediums, with an emphasis on oral and written communication to real-world audiences.
Abstractness	Students are required to focus on conceptual thinking within and across disciplines. They may examine the generalizations behind a specific concept, formulate their own generalizations, or move from concrete applications to more abstract ways of thinking about a concept or discipline.

Therefore, the content of the programme should require less drilling or memorization tasks, but focus on demonstrating how a small concept or a simple theorem can be generalized into a different theory of mathematics. Problems involved should be challenging enough and allow in-depth exploration. Moreover, the programme should demonstrate problems that can be solved using different approaches.

Robinson and Campbell (2010) suggested some practices particularly appropriate for gifted students.

- Teachers can set a fast learning pace, devoting little time to reinforcement in lessons.
- Teachers can assume high levels of motivation and good behavior in students.
- Teachers can engage in co-constructing knowledge with their students rather than, of in addition to, transmitting it.
- Teachers can encourage student self-assessment and meta-cognition for independent learning.

The above list can be the guidelines for teachers to adopt suitable strategies and lesson design. Teachers can act as facilitators to engage students in discovering knowledge and co-constructing new knowledge. The lesson can be designed to arouse and sustain the learning interest among students. Important self-learning skills and resources can be provided so that students can extend their learning beyond classrooms.

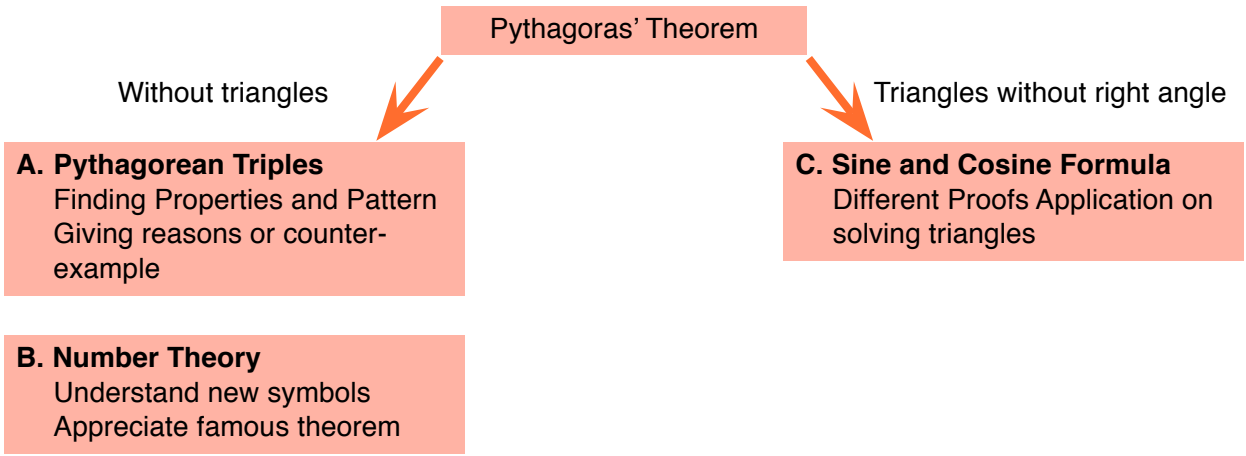
Learning and Teaching Strategies

An interactive approach is preferred so as to enliven the learning atmosphere and to arouse active discussions amongst students, instead of relegating them to be merely passive learners. During the pull-out programme, students could mainly work in groups formed according to their mathematical ability and relationship. This allowed students to exchange ideas and stimulate one another's thoughts. Teachers could assign students to lead the whole-class discussion on some of the content.

Since the students selected were interested in mathematics and had good foundations, pre-lesson tasks were given to them to prepare for the learning. In the beginning of each lesson, students were chosen to present their pre-lesson work. The content of the lesson was mainly presented through a guided discovery approach. With the scaffolding provided in the worksheets, students were encouraged to discover new knowledge or to prove new results by themselves. For some problems, multiple solutions were provided to students to deepen their understanding. To extend students' learning outside the classroom, extension materials involving internet resources were provided at the end of each unit.

Learning Content and Activities

This pull-out programme served as an extension of daily learning. The learning content was developed from Pythagoras' Theorem, which is usually taught in the S2 regular curriculum, in two different ways. One way was to explore the theorem about triangles without right angle. Another way was to investigate the integral solution of equations like ' a2 + b2 = c2 ' without the context of triangles. The following figures summarize the learning content and the highlighted mathematical thinking skills of the programme, which can be divided into 3 learning units as shown in the table. Each unit requires 60 minutes to 90 minutes depending on the students' ability and the depth of the discussion.



Unit	Topic	Pre-lesson Tasks	Learning Content/ Activities	Extension Materials
A	Pythagorean Triples	- Use of Python to generate 1000 Pythagorean triples - Pattern of Pythagorean Triples	- Formula about the triples - Inquiry tasks on patterns of triples	- 60-base system - Complex number
B	Number Theory	- Conditions for divisibility	- Introduction of Number Theory - Inquiry tasks on theorems in number theory - Story of Fermat's Last Theorem	- Basic theorem of congruence modulo - Proof about Irrational number
C	Cosine Formula	- Sine formula and area formula of triangle	- Limitation of Pythagoras' Theorem - Derivation of Cosine Formula - Solve a scalene triangle	- Other proofs of Cosine Formula

The learning content were designed based on the theoretical framework about a curriculum for the gifted.

Features	Related Content
Acceleration	Unit C includes a topic in the S5 mathematics curriculum due to its relation with Pythagoras' Theorem. Unlike a regular curriculum, students learn this advanced topic focusing on different proofs of the formula rather than doing drilling practice.
Complexity	The learning tasks require students to apply different mathematical concepts and thinking skills. For example, to study the proof of cosine formula in Unit C, students need to apply knowledge about Pythagoras' Theorem, coordinate geometry and trigonometry. Moreover, most tasks in Unit B expect students to generate knowledge by themselves.
Depth	
Challenge	In most parts of the programme, students learn the content through problem-solving or guided discovery. All these problems in the programme require strong reasoning skills. In Unit B, students might find it challenging as they face new symbols and theorems in mathematics. In order to solve the problems, they also need to relate their knowledge about integers to the new theorems, look for patterns and express patterns with the new symbols.
Abstractness	In Unit A, students can see the importance of observing patterns, making judgments by proof or giving counter-examples in mathematics inquiry. Throughout the programme, students can also see how Pythagoras' Theorem can be generalized to important results in other disciplines of mathematics.

Discussion

The programme was conducted as part of the S2 mathematics enrichment programme in the school. During the lesson, students were observed to have commitment in solving the challenging tasks and constructing mathematics knowledge. They did not give up easily and were willing to try. Some of them showed strong interest in the topics discussed in the lesson and the extension materials. However, students were not so active in group interaction and presentation. Teachers thought that this was because students were selected from different classes and hence needed some time to get used to the new classroom setting.

In the school, pre-lesson tasks and extension materials were provided as self-learning materials. Teachers may also have in-depth discussions about them within the lesson. If needed, teachers may separate each unit into more than one lesson. The focus of the programme is not just to solve the problem stated in the worksheets, but also to encourage students to make observations, judgements and generalizations. Teachers are encouraged to stimulate students' thought by asking more open ended questions like 'What else can you observe?', 'How can we prove it?', 'What else can you think of?'… The learning materials presented can only demonstrate some basic and introductory knowledge about the topics. Teachers are highly encouraged to enrich the units with related resources when adopting this exemplar.