Lesson Plan

Unit A

Prior Knowledge	- Pythagoras' Theorem - Algebraic Identities about Squares
Learning Objectives	- Students can relate the triples with the square identities - Students observe and describe some properties of the triples - Students can justify whether a Mathematical statement is true or not with algebraic manipulations - Students can justify whether a Mathematical statement is true or not with counter-examples - Students develop better number sense
Intended Learning Outcomes	- Students demonstrate rigorous Mathematical reasoning, writing down step-by-step algebraic proof instead of just an example, a counter-example to disprove a statement, and their thoughts with suitable Mathematical symbols and words - Students discover the properties of Pythagorean triples and get the correct answers for most of the questions on the worksheets - Students will discuss or ask questions when they are in doubt - Students present their findings to the whole class with confidence and clear voice
Learning & Teaching Strategies	Guided Discovery Activities, Presentation

Pre-lesson Tasks

1. Students use computer programming to generate a list of Pythagorean triples.
2. Students complete the proof of a property about Pythagorean triples.

Procedure

Learning Focus	Activity / Content	Learning & Teaching Strategies
Introduction	1. Review of Pythagorean Theorem and introduction of Pythagorean triples. 2. Students present their solution of Task 2 of Pre-lesson Worksheet 1.	Pre-lesson Worksheet 1
Generating formula of Pythagorean triples	1. Discussion about the generating formula of Pythagorean triples: $A=m^2-n^2$, $B=2mn$, $C=m^2+n^2$. 2. Students try a few questions in an online quiz[1]. 3. Students prove that the above substitution satisfies $A^2+B^2=C^2$. 4. Teacher can provide further knowledge about the generating formula depending on the level of the students.	Tablet for the online quiz
Property of Pythagorean triples	1. Teacher explains the definition of "Primitive Pythagorean Triples". 2. Students work in groups to explore the properties of the triples and answer questions in Lesson Worksheet 1. 3. Students share their findings.	Lesson Worksheet 1
Summary and extension	1. Teacher summarizes some important skills, including the use of algebraic method to prove a statement and giving a counter-example to disprove a statement. 2. Teacher introduces the extra materials and explain how the materials are related to the lesson. Students are encouraged to visit the websites.	Lesson Worksheet 1

[1] https://www.mathsisfun.com/numbers/pythagorean-triples.html

Extended Learning Activities

The following links are provided to students for further learning.

1. Website : Why We Still Use Babylonian Mathematics and the Base 60 System[2]
2. Video : Introduction to complex numbers[3]
3. Video : Finding Pythagorean triples from square of Complex numbers[4]

[2] https://www.thoughtco.com/why-we-still-use-babylonian-Mathematics-116679

[3] https://www.youtube.com/watch?v=SP-YJe7Vldo

[4] https://www.youtube.com/watch?v=QJYmyhnaaek

Lesson Plan

Unit B

Prior Knowledge	Basic concepts about factors, multiples and prime
Learning Objectives	- Students can develop better number sense - Students understand some basic knowledge in number theory such as Divisibility, Prime Factorization, Congruence etc. - Students carry out inquiry-based activity in number theory - Students appreciate the hard work and persistence of Mathematicians
Intended Learning Outcomes	- Students can comprehend unfamiliar Mathematical symbols and theorems, by reading and finish the simple questions and task about congruence - Students are committed to solving the problems, by trying even if they cannot get the answers immediately, and asking teaching for hints - Students will discuss or ask questions when they are in doubt - Students show interest about the story and history of Mathematical theorems and pay attention on presentation
Learning & Teaching Strategies	Guided Discovery Activities, Presentation

Pre-lesson Task

Students finish Pre-lesson Worksheet 2 about Divisibility Rule.

Procedure

Learning Focus	Activity / Content	Learning & Teaching Strategies
Divisibility Rule	1. Teacher introduces some basic knowledge such as Divisibility and Decimal number system. 2. Students present their solution of Challenges 1,2 and 3 of Pre-lesson Worksheet 2.	Pre-lesson Worksheet 2
Formula for number of factors	1. Students work in groups to finish the Inquiry Activity 1 on Lesson Worksheet 2. 2. Students share their findings.	Lesson Worksheet 2
Congruence and Fermat's Little Theorem	1. Students work in groups to complete the Inquiry Activity 2 on Lesson Worksheet 2. 2. Teacher go through some parts to make sure students have the basic understanding about congruence after reading the first few sections of the worksheet. 3. Students share their findings.	Lesson Worksheet 2
Story about Fermat's Last Theorem	Teacher share the story about Fermat's Little Theorem.	
Summary and extension	Teacher summarizes some important skills of observing number patterns and relations, generalizing observation into a theorem, and understanding the definition of new Mathematical symbols.	

Extended Learning Activities

The following links are provided for further learning. Teacher introduces the extra materials and explains how the materials are related to the lesson.

1. Prove that square root 2 is irrational: Making sense of irrational numbers[5] / The 5 Best Proofs that the Square Root of 2 is Irrational[6]
2. Fermat's little theorem visualization[7]
3. Fermat's little theorem examples[8]

[5] https://www.youtube.com/watch?v=sbGjr_awePE

[6] https://www.youtube.com/watch?v=IzkCVzzHHbg

[7] https://www.youtube.com/watch?v=OoQ16YCYksw

[8] https://www.youtube.com/watch?v=pMA-dD-KCWM

Lesson Plan

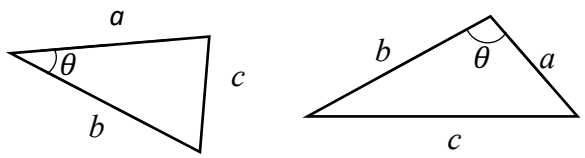
Unit C

Prior Knowledge	- Pythagoras' Theorem - Square Identities - Trigonometric Ratios
Learning Objectives	- Students can solve triangle with enough given information - Students understand a few proofs of Cosine Formula - Students can relate Cosine Formula with Pythagoras Theorem - Students appreciate the generalization from Pythagoras Theorem to Cosine Formula
Intended Learning Outcomes	- Students can comprehend unfamiliar Mathematical symbols and theorems, and understand new definition of trigonometric functions quickly - Students are committed to solving the problems, by trying even if they cannot get the answers immediately, and asking teaching for hints - Students demonstrates strong algebraic skills, by correctly expanding and simplifying expressions, and apply trigonometric identities like $(\sin^2\theta+\cos^2\theta=1)$ when simplifying expressions
Learning & Teaching Strategies	Guided Discovery Activities, Presentation

Pre-lesson Task

Students finish the Pre-lesson Worksheet 3 about Sine Formula and Area of Triangle.

Procedure

Learning Focus	Activity / Content	Learning & Teaching Strategies
Introduction	1. Teacher asks questions to check students' understanding about the pre-lesson knowledge of Sine Formula and Area Formula. 2. Teacher introduces the topic using the last question in the Pre-lesson Worksheet 3: If the three sides of a triangle are given, but none of the angles are given, can you still find out the angles? 3. Students are expected to point out the limitation of Sine Formula and the need for a new formula.	Pre-lesson Worksheet 3
Basic observation	1. Students are asked to draw an acute-angled triangle and an obtuse-angled triangle, denote a,b,c,θ as shown, and measure the lengths of a, b and c.  2. Students are asked to discuss whether $a^2 + b^2$ and c^2 still have specific relation in the two triangles 3. Teacher guides the students to summarize the following relation: For acute angle θ, $a^2 + b^2 > c^2$ For right angle θ, $a^2 + b^2 = c^2$ (Pyth. Theorem) For obtuse angle θ, $a^2 + b^2 < c^2$	
Basic knowledge	1. Students may see the trigonometric function (of angles greater than 90°), which is not learnt in S2 and S3 syllabuses. 2. Teacher can briefly explain the definition of trigonometric function and identities like $\cos(180° - \theta) = -\cos\theta$.	
Proofs / deduction of Cosine Formula	3. Students work in groups to complete Lesson Worksheet 3 about a few proofs of Cosine Formula. - Using the distance formula - Using trigonometry - Using the Pythagorean theorem 4. Students share their findings.	Lesson Worksheet 3
Practice questions	Students work individually to practice the use of sine formula, cosine formula and area formula.	Lesson Worksheet 4

Learning Focus	Activity / Content	Learning & Teaching Strategies
Origami activity	Teacher leads the students to prove the cosine formula by origami method.	Origami materials
Summary and extension	Teacher summarizes that the Cosine Formula is a generalization of Pythagoras Theorem. Pythagoras Theorem is a particular case of Cosine Formula when the angle is a right angle. This kind of generalization is common in Mathematics study.	

Extended Learning Activity

An extension worksheet is provided to show how to prove cosine formula using sine formula.