Unit C

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Unit C Cosine Formula Origami Materials

We can also prove the Cosine Formula by calculating areas. The change of sign as the angle γ becoming obtuse makes a case distinction necessary.

Recall that a^2, b^2 and c^2 are the areas of the squares with sides a, b and c respectively;

- if γ is acute, then $ab\cos\gamma$ is the area of the parallelogram with sides a and b forming an angle of $\gamma' = 90° - \gamma$;
- if γ is obtuse, and so $\cos\gamma$ is negative, then $-ab\cos\gamma$ is the area of the parallelogram with sides a and b forming an angle of $\gamma' = \gamma - 90°$.

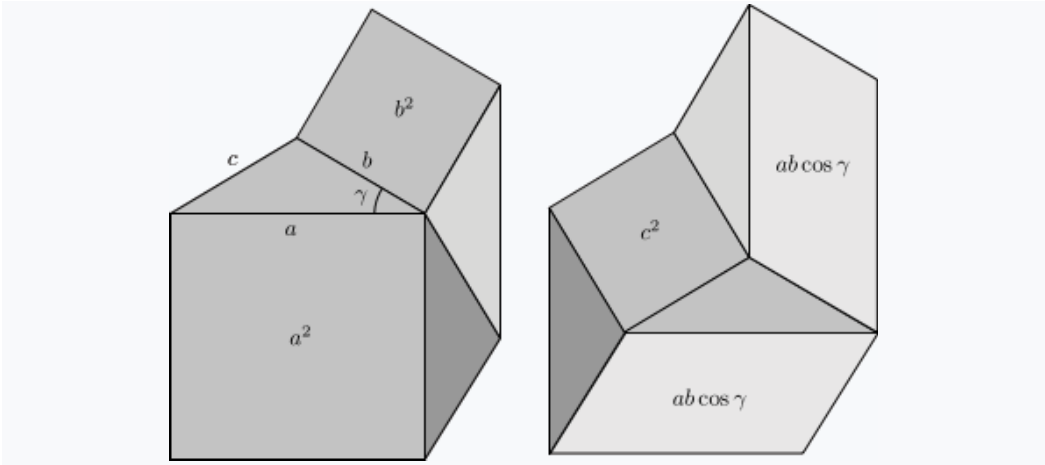

Fig. 6a – Proof of the Cosine Formula for acute angle γ by "cutting and pasting".

Acute case. Figure 6a shows a heptagon cutting into smaller pieces (in two different ways) to yield a proof of the Cosine Formula. The various pieces are

- in pink, the areas a^2, b^2 on the left and the areas $2ab\cos\gamma$ and c^2 on the right;
- in blue, the triangle ABC, on the left and on the right;
- in grey, auxiliary triangles, all congruent to ABC, an equal number (namely 2) both on the left and on the right.

The equality of areas on the left and on the right gives

$$a^2 + b^2 = c^2 + 2ac\cos\gamma$$

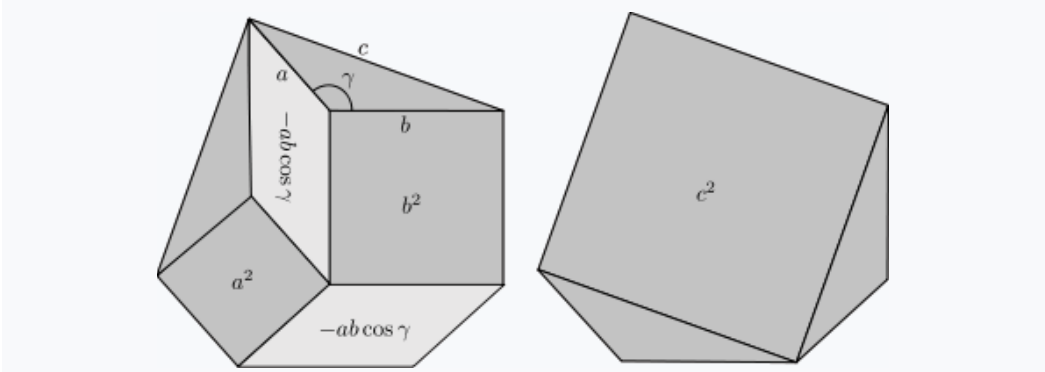

Fig. 6b – Proof of the Cosine Formula for obtuse angle γ by "cutting and pasting".

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Obtuse case. Figure 6b cuts a hexagon in two different ways into smaller pieces, yielding a proof of the Cosine Formula in the case that the angle γ is obtuse. We have

- in pink, the areas a^2, b^2 and $-2ab\cos\gamma$ on the left and c^2 on the right;
- in blue, the triangle ABC twice, on the left, as well as on the right.

The equality of areas on the left and on the right gives

$$a^2 + b^2 - 2ac\cos\gamma = c^2$$

The rigorous proof will have to include proofs that various shapes are congruent and therefore they have equal area. This will use the theory of congruent triangles (to proof?).

Reference: https://en.wikipedia.org/wiki/Law_of_cosines

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