賽馬會「知優致優」計劃
Jockey Club "Giftedness Into Flourishing Talents" Project

Proofs of Pythagoras' Theorem

Mathematics Secondary 2

Level 1: School-based Whole-class Teaching

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Background and Notes

The design of the learning and teaching plan reflects the actual circumstances of the particular school at the time of implementation. As it is developed and tailor-made to meet the specific cognitive and affective needs of students, all learning and teaching resources are for reference only.

When adapting the materials, curriculum, instructional and assessment modifications can be made in accordance with the diverse needs and abilities, learning styles and aspirations of students, professional competence of teachers, and gifted education development of the schools.

Teachers are strongly recommended to read the introduction, theoretical background and summary of the resource package to have a better understanding of the principles of Gifted Education and strategies for implementation.

This unit includes 1 foreword, 1 lesson plan, 8 sets of worksheets, 1 set of suggested answers and 3 set of extra materials.

With reference to our resources, educators can design suitable learning activities and implement the elements of Gifted Education, based on students' needs and interests, and teaching experience, so as to unfold students' potentials to the fullest.

Proofs of Pythagoras' Theorem

Grade: Secondary 2
No. of Lessons (Learning Time): 2 Consecutive Lessons (80 minutes)

Prior Knowledge	- Square Number and Square Root - Congruent and Similar Figures - Angle Relations about Straight Lines and Triangles - Algebraic Identities about Squares
Learning Objectives	- Students should be able to understand a few proofs of Pythagoras' Theorem - Students present the proofs of Pythagoras' Theorem - Students can apply Pythagoras Theorem to find unknown sides in a right-angled triangle - Students develop positive attitudes towards Mathematical proofs
Intended Learning Outcomes	- Students present Mathematical knowledge logically, fluently and systematically - Students follow the presentation attentively - Students solve complex tasks with various problem solving skills - Students demonstrate effective communication and collaboration skills - Students enjoy the process of exploring and sharing Mathematical knowledge
Learning & Teaching Strategies	Student-led Activity, Flexible Grouping, Tiered Assignment, E-learning
Operation Mode of Gifted Education	Level 1: School-based Whole-class Teaching
Core Elements of Gifted Education	Higher-order Thinking Skills Creativity Personal-social Competence

Foreword / Background

More often than not, rigorous Mathematical proofs are presented with a focus on logical reasoning and manipulative skills. This type of presentation may only appeal to students with a strong interest in Mathematics but not all students. Also, the proofs are usually delivered through a teacher-centred approach – the teacher describes the steps and students try to follow and understand the proof. However students who have yet to develop their interest and skills in math may not be fully engaged in the learning process, not to mention actually enjoying it. Moreover, given this approach, students with high Mathematical ability cannot stretch their potential by constructing the proofs by themselves.

The target group of this lesson was a class of about 30 students with mixed Mathematical ability. In terms of ability and prior knowledge in Mathematics, over half of them were of medium level while there were about 5 to 8 high-achievers and 3 to 5 low-achievers. In the past, the learning content were usually designed based on the medium-level students and the low-achievers as they were the majority of the class and needed more support from teachers.

Students had diverse interests and learning styles. About one-third of the students were talkative and liked to share ideas with others. The high-achievers loved to challenge themselves by solving complex problems. The low-achievers were less confident in learning Mathematics and needed step-by-step demonstrations when learning new formulae. Some students were visual learners and could learn better with the help of simulations or figures. The teacher found it difficult to cater for the diverse learning needs of the students.

Objectives of Collaboration

The aim of the collaboration was to design a student-centred lesson around the proofs of Pythagoras' Theorem based on students' potential and characteristics. Teachers rethought how to help students learn the proofs and hoped that students could enjoy learning Mathematical proofs and appreciate the wisdom behind the proofs. They also wanted to engage students with high Mathematical ability and those talkative students, make use of their strengths and help them stretch their potential.

Theoretical Framework

One of the key tasks of gifted education is to attend to the diverse potential and characteristics of individual students with a view to guiding and supporting them to develop their giftedness into flourishing talents (Education Bureau, n.d.).

According to the Theory of Multiple Intelligences (Gardner, 1983), there are eight intelligences embedded in the human mind, namely linguistic, logical, spatial, musical, bodily kinesthetic, intrapersonal, interpersonal, and spiritual, moral and existential intelligence intelligences. In the class, some students had high linguistic and interpersonal intelligences. These students had good presentation and communication skills. They liked to express their ideas verbally and work with others. There were also students with high visual/spatial potentials. They were interested in figures, simulations or origami. These students might not be high achievers in Mathematics. If the learning activities were designed to address their strengths and interests, they could be more engaged in the lesson.

To enhance the quality of education, three core elements of gifted education, namely higher-order thinking skills, personal-social competence and creativity, can be integrated into regular lessons. Constructing a Mathematical proof of a new theorem can help promote students' higher-order thinking skills as students need to relate the new theorem to their prior knowledge, the figures and simulations given and the guidelines provided by the teacher. It is a complex task which requires higher-order thinking skills such as reasoning and problem-solving skills. Students' personal-social competence could be developed through group activities and presentations. Suitable grouping provides students with a chance to work collaboratively and learn from peers. To prepare a presentation about Mathematics, students need to tidy up and organize abstract ideas. Through the process of formulating solutions, presenting ideas in their own ways and suggesting ways of improving their solutions, students' creativity can be developed and strengthened (Education Bureau, 2017).

In the Level 1 implementation of gifted education, differentiated teaching through appropriate grouping of students can be adopted to meet the variant needs of the groups resulting in enrichment and extension of the curriculum across all subjects in regular classrooms (Education Bureau, n.d.). In a differentiated classroom, learning activities, materials and products can be adjusted appropriately so that students with various needs can be appropriately challenged. The adjustments can be from foundational to transformational, concrete to abstract, simple to complex, single facet to multiple facet, small leap to great leap, structured to open-ended, dependent to independent or slow to fast (Tomlinson, 2005).

Teachers can adopt differentiation based on students' strengths, interests and learner styles. Learning materials can be adjusted and assigned according to students' Mathematical ability so that they are appropriately challenged. Students with high Mathematical ability can comprehend and remember Mathematics knowledge quickly. It is not challenging enough for them to just understand a theorem and solve routine problems in the textbook. Complex problems that involve higher-order thinking skills are suitable for them. For weaker students, more scaffolding can be provided so that they can accomplish the same learning objectives. For students who like to talk and share or those having good leaderships skills, learning tasks involving personal-social competence like group work or presentation can suit their interests and learner styles. Through differentiation, the teacher can utilize students' potential and create a chance for success for all students.

Learning and Teaching Strategies

One key learning objective of the lesson was that all students were expected to have a basic understanding of the Pythagoras' Theorem. Some basic components of a Mathematics lesson like demonstrating examples and practicing textbook questions were involved to help students master basic knowledge and skills. These components were essential for the less-able students when learning a new theorem.

The lesson also aimed to engage students with different learning needs in learning Mathematics and help them develop positive attitudes towards Mathematical proofs. To achieve these objectives, a student-led activity and two group activities with different ways of grouping and learning focus were designed.

1. Student-led activity

Seeing the high linguistic potential of some students, a student-led activity was designed. Two to three students with good presentation and interpersonal skills were chosen as student leaders. They were responsible for leading an origami activity that demonstrated a proof of Pythagoras' Theorem. Prior to the lesson, teachers introduced the origami activity to those students and asked them to prepare an interactive presentation. In the lesson, they were expected to state the Theorem accurately, lead the class with clear instructions, write down the algebraic steps correctly and raise questions to the whole class. These student leaders would have the chance to stretch their potential in linguistic, spatial, bodily-kinesthetic and interpersonal domains.

2. Group Activity 1 [Proof Exploration] : Homogeneous Grouping

To cater for learners with different levels of Mathematical ability, students were divided into to eight groups: 2 high-ability groups, 4 medium-ability groups and 2 groups mixing some medium-ability and some low-ability students. Eight sets of worksheets with different levels of complexity were assigned to the groups accordingly. In each group, students viewed GeoGebra material about a proof of the theorem and had to fill in the details of the proof.

For the high-ability groups, students needed to apply cross-topic knowledge and higher-order thinking skills to finish the tasks. Some of the tasks also involved theorems they had never learnt before. For the other groups, the worksheets were simpler or had more guidelines. Since high-ability groups were provided with more challenging tasks, they might spend time struggling or might ask for hints from teachers. For the other groups, as more guidelines were provided in the worksheets, students were expected to finish the tasks without help from teachers. Simpler proofs about Pythagoras' Theorem were assigned to the groups with mixed medium-ability and low-ability students. Low-ability students could receive help from the medium-ability students in the group. The above grouping design allowed teachers to spend more time with the high-ability groups and facilitate further discussion among them. After the activity, it was expected all students would have in-depth understanding of the proof assigned to them, which is one major learning objective of the lesson.

3. Group Activity 2 [Proof Presentation] : Heterogeneous Grouping

After exploring a proof, presentation and sharing tasks were then carried out using a mixed ability grouping. Students were grouped with classmates having different sets of worksheets. In the groups, students took turns to explain the proof they had worked with. Teachers walked around or sat in some groups to listen to the presentation and comment on their presentation skills. Some students could also be chosen to present to the whole class. In this activity, all students, even the students less able in Mathematics, were expected to share their findings with confidence and appreciate others' work. Talkative students might find this activity enjoyable. Through exchanging knowledge, students could broaden their horizons on the topic. As such, their personal-social competence could be enhanced.

A flexible grouping strategy was adopted in this lesson by having two group activities with different learning foci and grouping nature. According to Tomlinson (2005), a flexible grouping strategy allows students to work both with students most like themselves and with students dissimilar from themselves. When a learning task is designed, teachers could assign work groups based on students' characteristics and the objective of the tasks. The proof-exploration task aims to allow students to study a proof that matches their Mathematical ability. Therefore, groups with similar ability are more suitable. The proof-presentation task encourages students to exchange knowledge and nurtures students' presentation skills. Hence, heterogeneous grouping is more appropriate.

Appropriate assignments can allow students to explore ideas at a level that builds on their prior knowledge and prompts continued growth. Tiered assignments were designed for this lesson. Teachers can design tiered assignments by adjusting the task for complexity, abstractness, number of steps, concreteness and independence (Tomlinson, 2005). This strategy allows students to work with appropriately challenging tasks and promotes motivation.

Proofs of Pythagoras' Theorem are closely related to figures and diagrams. During the lesson, visual tools like computer simulations and hand-on activities like paper-cutting were involved in the lesson to suit the visual learners.

Given that student-centred learning activities addressed students' different potentials, interest and learning styles, students were expected to be more engaged in the lesson and develop a positive attitude towards Mathematical proofs. With the lesson materials differentiated by their learning needs, students of all levels can master the basic knowledge and also be appropriately challenged. Thus, the lesson could provide a chance for success for all students in the class, not only those with high Mathematical ability.

Discussion

The Project GIFT team and teachers who conducted the lesson observed that students were engaged in and committed to the learning process. Student leaders had high initiatives doing the preparation before the lesson. They demonstrated effective communication skills. They also had bright smiles during the presentation, which implied that they enjoyed their roles as presenters. This also showed that the lesson helped to establish students' personal-social competence. Their classmates followed their instructions and actively answered the questions posed by them. So students obtained a strong sense of satisfaction in this learning process while their personal-social competence was enormously enhanced. Moreover, students with high Mathematical ability benefitted from the more challenging tasks and the homogenous groupings. These groups of students received the most challenging worksheets. They could not figure out the answer at once, so they did not finish the task faster than the others and be left idle. Instead, they spent time discussing, looking for hints and finally solved the problem. Even the students weaker in Mathematics enjoyed the lesson. They did not find it embarrassing to receive an easier worksheet. Instead, they could finish with only a little help from teacher and they cherished the opportunities to present the proofs that other classmates had not yet studied. This appropriate level of the learning task leads to the development of students' high- order thinking. In the proof presentation activity, most students could present the Mathematics knowledge correctly. Some of them presented with fluent language and strong confidence.

Teachers who conducted the lesson found that they needed much time to prepare before the lesson. They needed to help student presenters to prepare and rehearse for the student-led activity. It also took much time for teachers to think about the hints and questions for eight different worksheets. However, teachers found these extra efforts worthwhile when they saw that even the students who were not interested in Mathematics in the past participated actively in the lesson.

To prepare for the student-led activity, teacher can choose two to three students with high linguistic and interpersonal potentials (need not be the high achievers in Mathematics) as the student leaders. Teacher are advised to meet these leaders a few times to clarify their understanding of the topic, to rehearse the student-led activity and to polish their presentation skills.

Before conducting the lesson, teachers are advised to arrange the first grouping based mainly on students' Mathematical abilities. Other characteristics of students should also be considered. For example, teachers may arrange the groups so that each group contains a student with good leadership skills. Or teachers may need to take care of the needs of SEN students when arranging the groups.

Teachers can adjust the level of challenge by revising the guidelines in the worksheets or selectively adapting some of the worksheets provided. It may be easier for teachers to handle if only four worksheets are chosen to be used. Each group could be given two worksheets with less challenging questions and followed by another two worksheets which are more challenging. Teachers can select the worksheets based on the Mathematical ability of the class. For example, teachers can select the more difficult worksheets if the class involves more students with high Mathematical ability.

In the lesson, teachers can mainly act as facilitators by providing hints and feedback to the groups.

Teachers can also summarize the learning focus and check students' understanding after each section of the lesson. It may be difficult to support all eight groups during the lesson. The teacher may prepare hint cards so students can read the hints by themselves. Alternatively, the school may arrange more than one teacher to prepare for and co-teach this lesson. To nurture collaboration skills, teachers can also add some reflection elements into the group activities. For example, they may ask students to comment on one another's performance in the group or may ask students to rate their own contribution to the group.

To conclude, this lesson illustrated some rationale and strategies to design a lesson that can make use of students' strengths, cater for diverse learning needs and stretch the potential of students with high ability. Teachers can also make use of these strategies to design lesson with topics other than proofs of Pythagoras' Theorem.