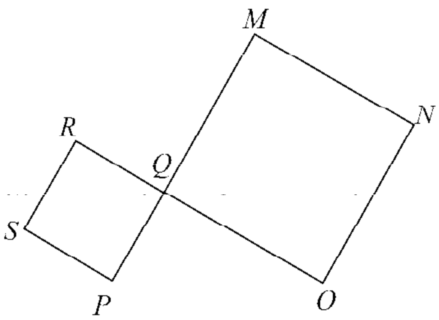
Lessons 1 -2

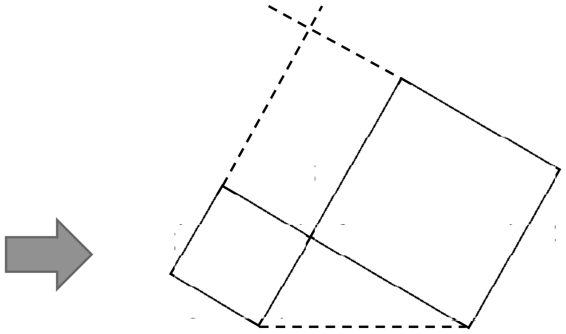
Suggest Answers and Guidelines

Name: ________________________ **Class:** _______ (   )

Set 1 --- Proof by Rearrangement [Solution]

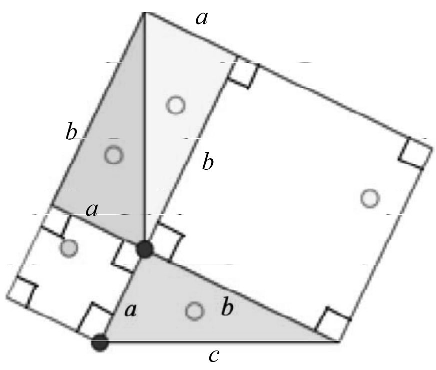


Two squares are put together with vertices P, Q and M collinear.



Lines are extended from the edge of the square or joined from the vertices

Figure 1 is then formed by cutting the top rectangle into two triangles. As shown.



Q1. Is $\angle PQO = 90^\circ$? Why?

$\angle PQR = 90^\circ$ and $\angle MQO = 90^\circ$ (square)
$\angle PQO = 180^\circ - 90^\circ = 90^\circ$ (adj. $\angle$s on st. line)

Q2. Are the three triangles congruent? Which reason (SSS, SAS, ASA, AAS, RHS) can explain the congruence?

Yes, SAS (the two sides with length a and b, the 90°.

Q3. Express the area of the whole figure in terms of a and b. [Hint: Three triangles + Two Square]

$$a^2 + b^2 + \frac{3ab}{2}$$

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Suggest Answers and Guidelines

Figure 1

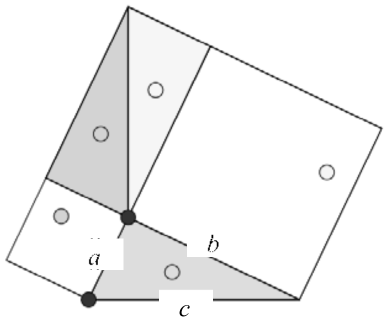
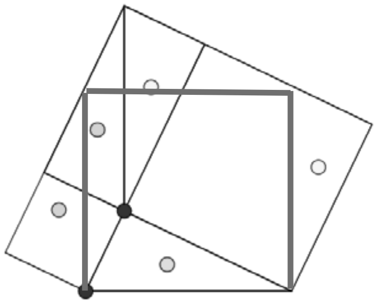


Figure 2



Q4. Following the GeoGebra, triangles are moved to new positions. Draw lines in **Figure 2** to show it.

Q5. Express the area of the whole figure in terms of a, b and c. [Hint: Three Triangles + One Square]

$$\boxed{c^2 + \frac{3ab}{2}}$$

Q6. Prove the Pythagoras' Theorem ($a^2 + b^2 = c^2$) using the areas in **Q3** and **Q5**.

$$\boxed{\begin{array}{c} a^2 + b^2 + \dfrac{3ab}{2} = c^2 + \dfrac{3ab}{2} \\ a^2 + b^2 = c^2 \end{array}}$$

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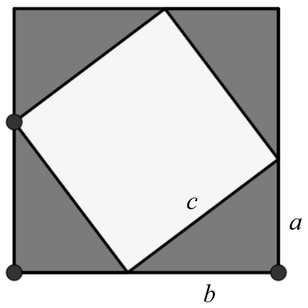
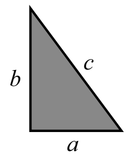
Suggest Answers and Guidelines

Name: ______________________ **Class:** _______ (   )

Set 2 --- Proof by Rearrangement [Solution]

Figure 1





Q1. How is Figure 1 formed?

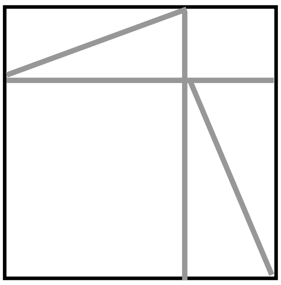
It is formed by rearranging four **congruent** triangles into a big **square** .

Q2. What kind of quadrilateral is formed in the yellow part?

A square (with length c)

**[Obviously the 4 lengths are all equal to c ,
Can ask student to prove the angles are 90°]**

Figure 2 Follow the slider in the Geogebra to draw the new figure obtained.



Q3. Two quadrilateral is formed in the yellow part now. What kind of quadrilaterals are they?

Squares.

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Suggest Answers and Guidelines

Name: ________________________ **Class:** _______ ( )

Set 3 --- Proof by Rearrangement [Solution]

At the beginning, two squares are placed side by side in **Figure 1**.

When you follow the slider in the GeoGebra, two white triangles are formed in **Figure 2**.

Figure 1

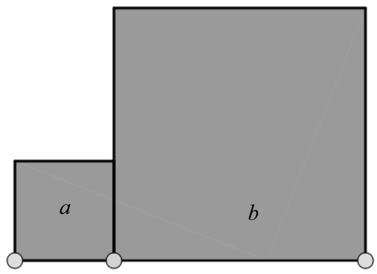
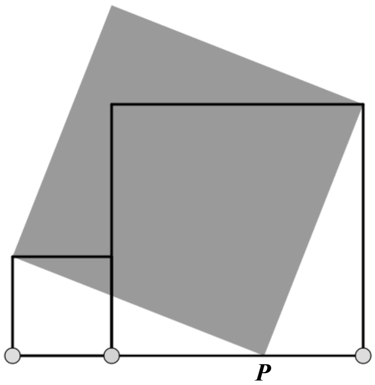
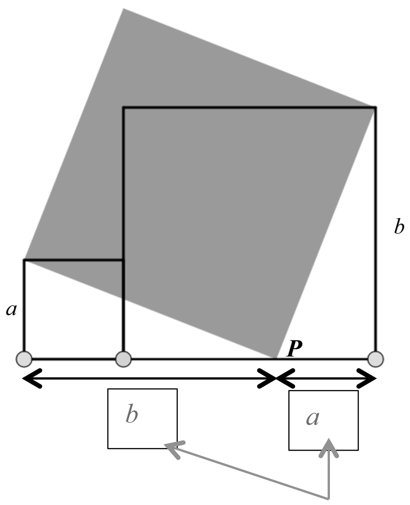


Figure 2



Move the green dot to change the size of the squares and observe the change in point P.

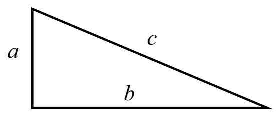
Figure 3



Question: Write down the length of these two line segments in terms of a and b.

Can you prove the Pythagoras' Theorem ($a^2 + b^2 = c^2$) using **Figure 1** and **Figure 3**?

Let a, b and c be the sides of the white triangles as shown.



In **Figure 1**, the purple parts are squares, the total area = $\boxed{a^2 + b^2}$

In **Figure 3**, the purple part is a square, the area = $\boxed{c^2}$

Therefore, $\boxed{a^2 + b^2 = c^2}$

Suggest Answers and Guidelines

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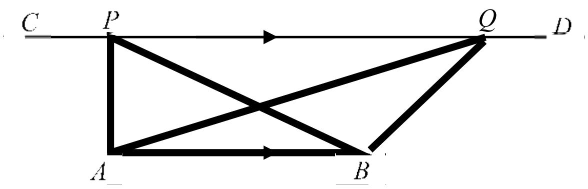
Set 4 --- Proof by Area Relation [Solution]

Observation: Given two parallel line segments AB and CD.

Two triangles APB and AQB are drawn with AB as their common base.



What is the relation of the **Area** of ΔAPB and ΔAQB?

Hint: Two triangles have the same base AB, how about the heights?

The height is the same. Therefore, the area are equal.

Following the slider in the Geogebra, Figures are obtained. Lengths a, b and c are marked as shown.

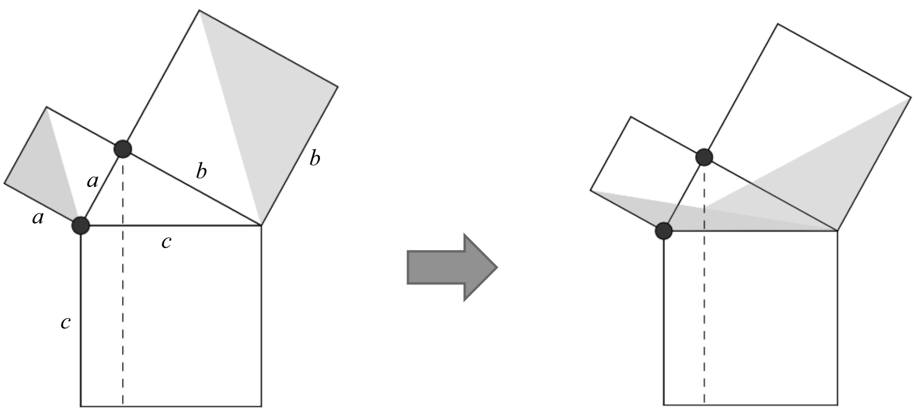


Figure 1 **Figure 2**

Q1. In **Figure 1**, find the area of the green triangle and blue triangles in terms of a and b.

Area of Green Triangle $= \boxed{\dfrac{a^2}{2}}$ Area of Blue Triangle $= \boxed{\dfrac{b^2}{2}}$

Q2. In **Figure 2**, find the area of the green triangle and blue triangles in terms of a and b.

(Hint: Use the **Observation**. How are the new triangles related to those in **Figure 1**?)

Area of Green Triangle $= \boxed{\dfrac{a^2}{2}}$ Area of Blue Triangle $= \boxed{\dfrac{b^2}{2}}$

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Suggest Answers and Guidelines

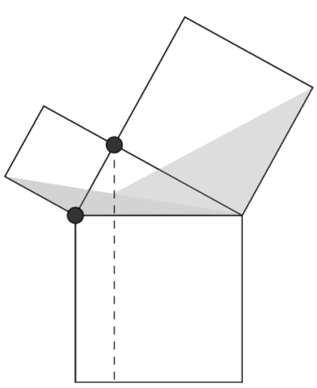


Figure 2

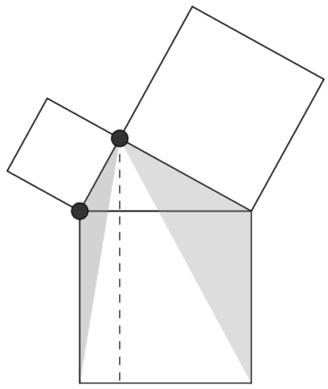


Figure 3

Q3. In **Figure 3**, the two triangles are transformed.

What kind of transformation is it? Will the area be changed after the transformation?

$\boxed{\text{Rotation}}$. The area (~~will~~ / will not) be changed.

Area of Green Triangle $= \boxed{\dfrac{a^2}{2}}$ Area of Blue Triangle $= \boxed{\dfrac{b^2}{2}}$

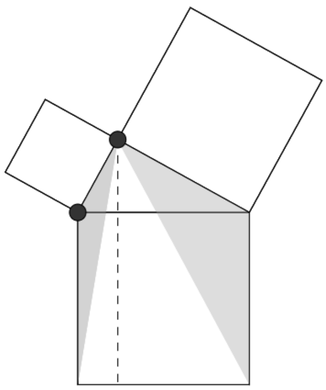


Figure 3

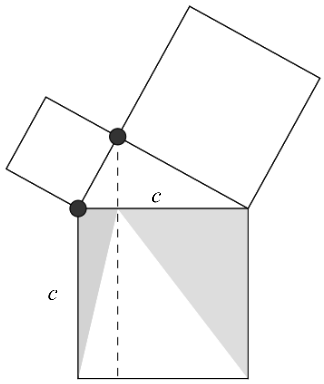


Figure 4

Q4. In **Figure 4**, find the area of the green triangle and blue triangle in terms of a and b.

(Hint: Use the **Observation**. How are the new triangles related to those in Figure 3?)

Area of Green Triangle $= \boxed{\dfrac{a^2}{2}}$ Area of Blue Triangle $= \boxed{\dfrac{b^2}{2}}$

Q5. In **Figure 4**, the two triangles are now inside the square with side c.

Find the sum of the two triangles in terms of c.

Area of Green Triangle + Area of Blue Triangle $= \boxed{\dfrac{c^2}{2}}$

(Hint: What is the fraction of the area of two triangles compared to the square with side c.)

Q6. Can you prove the Pythagoras' Theorem ($a^2 + b^2 = c^2$) using the area relation?

$$\boxed{\frac{a^2}{2} + \frac{b^2}{2} = \frac{c^2}{2} \Rightarrow a^2 + b^2 = c^2}$$

Lessons 1 -2

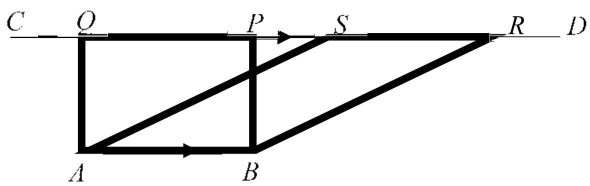
Suggest Answers and Guidelines

Name: ______________________ **Class:** _______ (

Set 5 --- Proof by Area Relation [Solution]

Observation: Given two parallel line segments AB and CD.

A rectangle $ABPQ$ and parallelogram $ABRS$ are drawn with AB as their common base.



What is the relation of the **Area** of rectangle $ABPQ$ and parallelogram $ABRS$?

(Hint: They have the same base AB. How about the heights?)

The height is the same. Therefore, the area is equal.

Following the slider in the Geogebra, Figures are obtained. Lengths a, b and c are marked as shown.

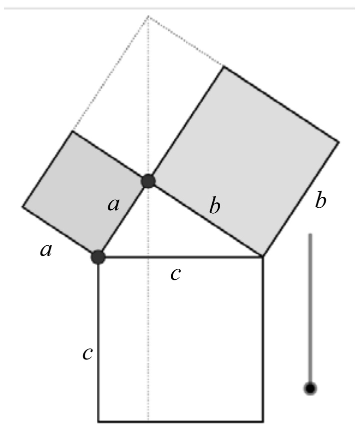


Figure 1

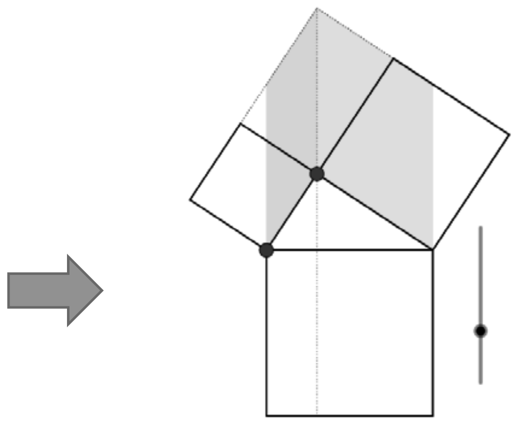


Figure 2

Q1. In **Figure 1**, find the area of the green part and blue part in terms of a and b.

Area of Green part $= \boxed{a^2}$ Area of Blue part $= \boxed{b^2}$

Q2. In **Figure 2**, find the area of the green parallelogram and blue parallelogram in terms of a and b.

(Hint: Use the **Observation**, how are the new parts related to those in **Figure 1**?)

Area of Green part $= \boxed{a^2}$ Area of Blue part $= \boxed{b^2}$

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Suggest Answers and Guidelines

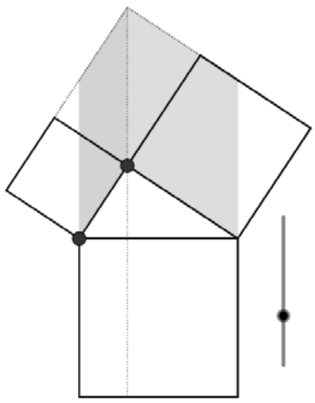

Figure 2

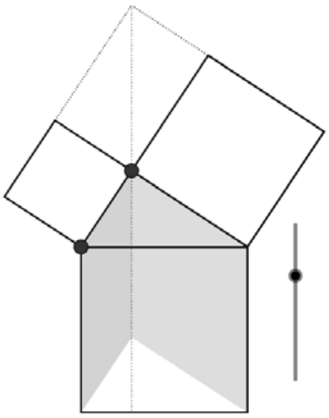

Figure 3

Q3. In **Figure 3**, the two parallelograms are transformed.

What kind of transformation is it? Will the area be changed after the transformation?

$\boxed{\textbf{Translation}}$. The area (~~will~~ / will not) be changed.

Area of Green part = $\boxed{a^2}$ Area of Blue part = $\boxed{b^2}$

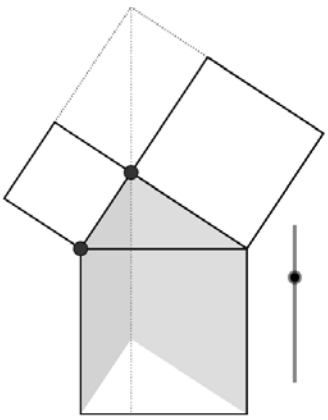

Figure 3

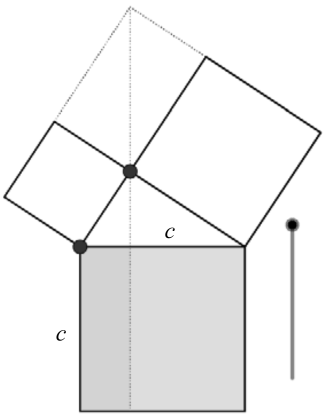

Figure 4

Q4. In Figure 4, find the area of the green rectangle and blue rectangle in terms of a and b.

(Hint: Use the **<u>Observation</u>**. How are the new triangles related to those in Figure 3?)

Area of Green part = $\boxed{a^2}$ Area of Blue part = $\boxed{b^2}$

Q5. In Figure 4, find the sum of the two rectangles in terms of c.

Area of Green Triangle + Area of Blue Triangle = $\boxed{c^2}$

Q6. Can you prove the Pythagoras' Theorem ($a^2+b^2=c^2$) using the area relation?

$\boxed{a^2+b^2=c^2}$

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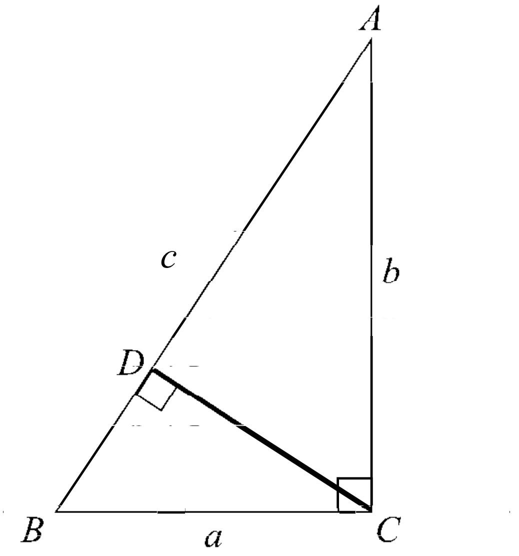
Suggest Answers and Guidelines

Name: ________________________ **Class:** _______ (   )

Set 6 --- Proof by Similar Triangles [Solution]



Let $AB = c$

Q1 What are the relations of the three triangles ΔBAC, ΔBCD and ΔCAD?

They are $\boxed{\textbf{similar}}$ triangles.

Q2 Following the steps below, prove the Pythagoras' Theorem .

Step.1 Using $\Delta BAC \sim \Delta BCD$

$$\frac{BD}{BC} = \frac{BC}{BA}$$

$$\frac{BD}{a} = \frac{a}{c}$$

$$BD = \frac{a^2}{c}$$

Step.2 Using $\Delta BAC \sim \Delta CAD$

$$\frac{DA}{CA} = \frac{CA}{BA}$$

$$\frac{DA}{b} = \frac{b}{c}$$

$$DA = \frac{b^2}{c}$$

Step. 3 $c = BD + DA$

$c =$ ——— + ———

$c = \frac{\qquad}{c}$

$$\therefore c^2 = a^2 + b^2$$

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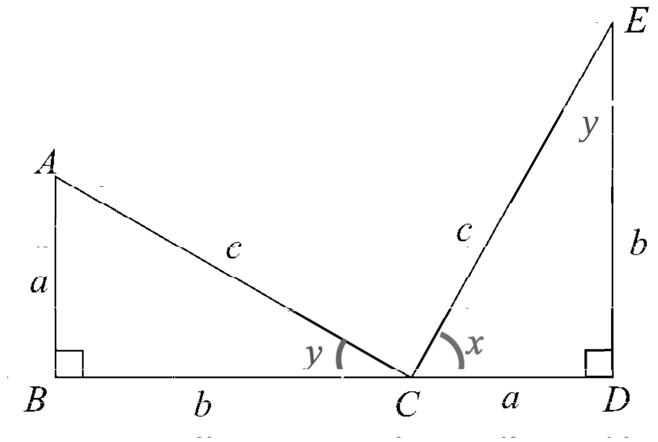
Suggest Answers and Guidelines

Name: ______________________ **Class:** _______ (   )

Set 7 --- Garfield's proof [Solution] ***Garfield is the President of US in 1881.**



Draw a line joining A and E. A trapezium $ABDE$ is formed.

Q1 Is $\angle ACE = 90^\circ$? Why?

$$
\boxed{\begin{aligned}
x + y + 90^\circ &= 180^\circ \ (\angle \text{ sum of } \Delta) \\
x + y &= 90^\circ \\
x + y + \angle ACE &= 180^\circ \ (\text{adj. } \angle\text{s of st. line}) \\
90^\circ + \angle ACE &= 180^\circ \\
\angle ACE &= 90^\circ
\end{aligned}}
$$

Q2 Following the steps below, prove the Pythagoras' Theorem ($a^2 + b^2 = c^2$).

Step.1 Area of Trapezium $ABDE$ = $\boxed{\frac{1}{2}(a+b)(a+b) = \frac{1}{2}(a+b)^2}$

Step.2 Area of ΔABC + Area of ΔCDE + Area of ΔACE (in terms of a , b and c)

$= \boxed{\frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}(c^2)}$

Therefore, $\frac{1}{2}(a+b)^2 = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}(c^2)$

$$(a+b)^2 = ab + ab + (c^2)$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

$$a^2 + b^2 = c^2$$

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(**Set 8 --- Proof by Similar Triangles** **[Solution]**

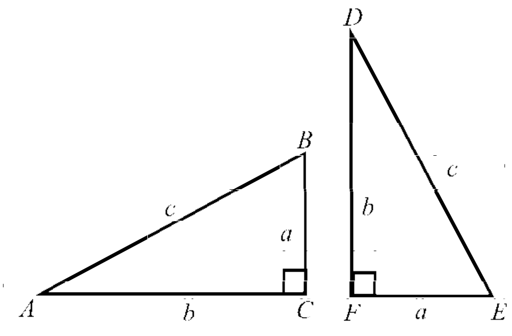

Figure 1

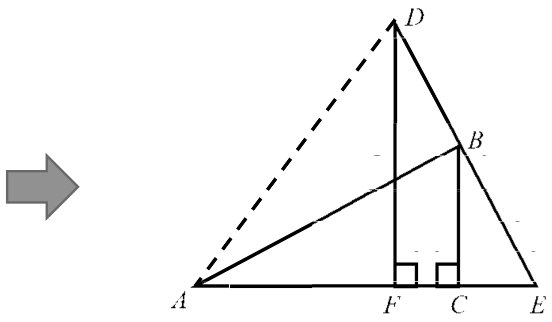

Figure 2

Two congruent right-angled triangles *ABC* and *DEF* are shown. (Figure 1)

They are then placed such that *B* lies on *DE* and *AFCE* forms a straight line. (Figure 2)

Step 1 In Figure 2, $DF \parallel BC$ (Reason: $\boxed{\textbf{int. } \angle\textbf{s supp.}}$)

Step 2 In Figure 2, Prove that $\Delta BCE \sim \Delta DFE$.

$$\angle BCE = \angle DFE = 90^\circ \quad (given)$$

$$\angle BEC = \angle DEF \quad (common\,\angle \qquad)$$

$$\angle CBE = \angle FDE \quad (3rd\ \angle \qquad)$$

$$\therefore \Delta BCE \sim \Delta DFE \ (\ A.A.A.\ )$$

Step 3 Express the area of ΔADE in terms of c .

Notice that $\angle ABE = 90^\circ$

Area of $\Delta ADE = \dfrac{1}{2} \times DE \times AB = \dfrac{c^2}{2}$

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Suggest Answers and Guidelines

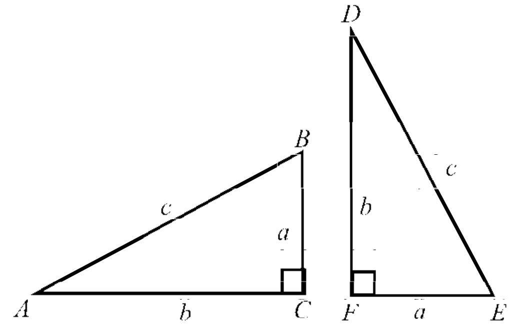

Figure 1

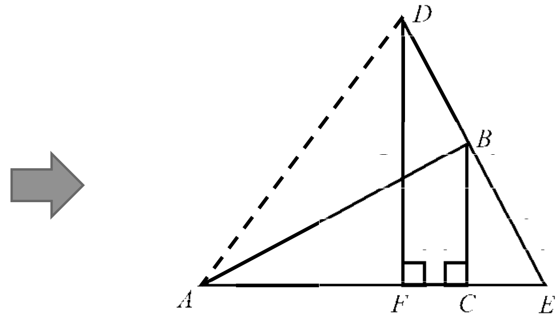

Figure 2

Step.4 Express the area of ΔADE in terms of a and b .

$\because \Delta BCE \sim \Delta DFE$,

$$\frac{CE}{EF} = \frac{BC}{DF}$$

$$\frac{CE}{a} = \frac{a}{b}$$

$$CE = \frac{a^2}{b}$$

$$\begin{aligned}\text{Area of } \Delta ADE &= \frac{1}{2} \times AE \times DF \\ &= \frac{1}{2} \times (AC + CE) \times DF \\ &= \frac{1}{2} \times (b + \frac{a^2}{b}) \times b \\ &= \frac{1}{2}\left(\frac{b^2 + a^2}{b}\right) \times b \\ &= \frac{1}{2}(b^2 + a^2)\end{aligned}$$

Combining steps 3 and 4 , prove the Pythagoras' Theorem ($a^2 + b^2 = c^2$) .

$$\frac{c^2}{2} = \frac{1}{2}(b^2 + a^2)$$

$$c^2 = b^2 + a^2$$