

Lesson Plan

Unit A, B & H

Lesson plans are not provided. Teacher can refer to regular curriculum content and extension materials.

Unit C

Topic	Triangle and Rectangle
Prior Knowledge	<ul style="list-style-type: none">- Basic Trigonometry- Completing Square- Heron's Formula
Learning Objectives	<ul style="list-style-type: none">- Students can develop intuitive idea about maximizing area- Students can apply algebraic method in Mathematical inquiry- Students can choose suitable formula in Mathematical inquiry
Intended Learning Outcomes	<ul style="list-style-type: none">- Students investigate the problems with algebraic method, including setting up variables according to the question setting, choosing appropriate formula to calculate the area of triangle, distinguishing the constant from variables in an expression, and describing the geometric meaning implied by the algebraic expressions- Students are committed to solving the problems, by trying even if they cannot get the answers immediately, and asking teaching for hints
Learning & Teaching Strategies	Guided Discovery Activities, Group Discussion

Pre-lesson Task

Visit the websites and finish Pre-lesson Worksheet 1:

<https://www.mathsisfun.com/algebra/trig-sin-cos-tan-graphs.html>

<https://www.youtube.com/watch?v=a0LvqflQMx4>

Procedure

Learning Focus	Activity / Content	Learning & Teaching Strategies
Review on the prior knowledge	Teacher makes a review on the prior knowledge related to the unit with the students.	
Pre-lesson knowledge	Students share their pre-lesson findings with reference to the Pre-lesson Worksheet 1.	Pre-lesson Worksheet 1
Problem-solving	Students work in groups to solve the three problems in the Lesson Worksheets. Teachers can provide hints when necessary. Students then present their solutions.	Lesson Worksheets 1-3
Summary	<ol style="list-style-type: none"> 1. Teacher summarizes some important inquiry skills, including choosing suitable formula, and the algebraic method of setting variables and constants, completing the square etc. 2. Teacher introduces the isoperimetric problem – A circle encloses the greatest area of a closed curve under a fixed perimeter. 	

Unit D

Topic	From Triangle to Quadrilateral
Prior Knowledge	Congruent Triangles
Learning Objectives	<ul style="list-style-type: none"> Students understand that a statement can be disproved by counter-examples Students appreciate that Mathematics results can be generalized to get new problems and new findings
Intended Learning Outcomes	<ul style="list-style-type: none"> Students can give counter-example to disprove statements Students are committed to solving the problems, by trying even if they cannot get the answers immediately, and following teachers' guidelines to move forward
Learning & Teaching Strategies	Guided Discovery Activities, Group Discussion

Pre-lesson Task

Visit the websites and finish Pre-lesson Worksheet 2:

<https://www.basic-mathematics.com/triangle-inequality-theorem-proof.html>

<https://www.quora.com/How-do-I-prove-that-sum-of-any-three-sides-in-a-quadrilateral-is-greater-than-the-fourth-side>

Procedure

Learning Focus (Time)	Activity / Content	Learning & Teaching Strategies
Pre-lesson knowledge	Students present the proofs of Triangle Inequality and Quadrilateral Inequality	Pre-lesson Worksheet 2
Problem-solving	Students work in groups to solve problems in the Lesson Worksheets. Teachers can provide hints when necessary. Students then present their solutions.	Lesson Worksheets 4 - 5
Summary and extension	<ol style="list-style-type: none"> Teacher summarizes some important skills, including giving counter-example to disprove a statement, and generalizing results may lead to new problems and eventually new findings Teacher can introduce the four special centres of triangles and the related geometric properties. Teachers can also ask students to pose question about quadrilateral based on some property of triangles. 	

Extended Learning Activity

Prove that the three angle bisectors of triangle are concurrent. Students can visit the website for the solution and more information.

<https://www.algebra.com/algebra/homework/Triangles/Angle-bisectors-of-a-triangle-are-concurrent.lesson>

Unit E

Topic	Property of Circle
Prior Knowledge	Basic Geometry
Learning Objectives	<ul style="list-style-type: none"> - Students can understand the idea of Proof by Contradiction - Students can separate the cases appropriately when proving a statement
Intended Learning Outcomes	<ul style="list-style-type: none"> - Students can understand the idea of Proof by Contradiction, by writing down correctly the negation of an 'If-Then' statement, and constructing suitable figures and adding suitable lines related to the negation - Students are committed to solving the problems, by trying to sketch figures to get ideas about the problems, trying to construct different quadrilaterals and look for pattern, and asking teaching for hints
Learning & Teaching Strategies	Guided Discovery Activities, Group Discussion

Pre-lesson Tasks

Visit the websites and finish Pre-lesson Worksheet 3:

<https://revisionmaths.com/gcse-maths-revision/shape-and-space/cyclic-quadrilaterals>

Procedure

Learning Focus	Activity / Content	Learning & Teaching Strategies
Pre-lesson knowledge	<ol style="list-style-type: none"> 1. Students share their pre-lesson findings with reference to the Pre-lesson Worksheet 3. 2. Teacher summarizes that sometimes we need to separate the cases when proving a statement 	Pre-lesson Worksheet 3
Problem-solving	Students work in groups to solve the problems. Teachers can provide hints when necessary (Outlining a proof by contradiction may be new and difficult to students. Teacher may provide more help). Students then present their solutions, and teacher restates the logic of Proof by Contradiction.	Lesson Worksheets 6 - 7
Summary and extension	<ol style="list-style-type: none"> 1. Teacher summarizes some important inquiry skills, including separating proof into cases, and Proof by Contradiction. 2. Teacher introduces some more proofs by contradiction: <ul style="list-style-type: none"> - There are infinitely many primes - $\sqrt{2}$ is irrational - Converse of Pythagorean Theorem 3. Teacher can ask students to try outlining a proof by contradiction to write down the negation of the above statements. 	

Extended Learning Activity

The following proof by contradiction can be introduced to students.

- A Proof for the Converse of the Pythagorean Theorem¹
- Making sense of irrational numbers - Ganesh Pai – A proof by contradiction is included in the video²
- Proof: There are Infinitely Many Primes (There is no Largest Prime)³

¹ <https://jwilson.coe.uga.edu/emat6680/brown/6690/ConPythagThm.htm>

² https://www.youtube.com/watch?v=sbGjr_awePE

³ <https://www.youtube.com/watch?v=ZYkZws-23R8>

Unit F

Topic	Polygon (1)
Prior Knowledge	<ul style="list-style-type: none"> - Basic Geometry - Proof by Contradiction
Learning Objectives	<ul style="list-style-type: none"> - Students can understand the idea of 'Proof by Giving Geometric Construction' - Students can apply Proof by Contradiction
Intended Learning Outcomes	<ul style="list-style-type: none"> - Students can understand the idea of "Proof by Giving Geometric Construction", by suggesting (one or more) ways to convert a concave polygon to convex polygon with the same perimeter, and explaining the construction steps using straight edge and compasses - Students can apply the proving skills flexibly, using Proof by Contradiction and writing down the negation of the statement in the problems - Students are committed to solving the problems, by trying even if they cannot get the answers immediately, and asking teaching for hints
Learning & Teaching Strategies	Guided Discovery Activities, Group Discussion

Pre-lesson Task

Finish Pre-lesson Worksheet 4.

Procedure

Learning Focus (Time)	Activity / Content	Learning & Teaching Strategies
Pre-lesson knowledge	<ol style="list-style-type: none"> 1. Students share their pre-lesson findings with reference to the Pre-lesson Worksheet 4. 2. Teacher summarizes that sometimes we need to separate the cases when proving a statement. 	Pre-lesson Worksheet 4
Problem-solving	Students work in groups to solve the problems in the Lesson Worksheets. Teachers can provide hints when necessary. Students then present their solutions, and teacher restates the logic of Proof by Contradiction and "Proof by Giving Geometric Construction".	Lesson Worksheets 8 - 9
Summary and extension	<ol style="list-style-type: none"> 1. Teacher summarizes some important inquiry skills, including Proof by Contradiction and "Proof by Giving Geometric Construction". 2. Teacher introduces some more basic construction skills using straight edge and compasses. 	

Extended Learning Activity

Refer to Problem (H) on Lesson Worksheet 8, give the detailed steps of the geometric constructions using straight edge and compasses.

Unit G

Topic	Polygon (2)
Prior Knowledge	<ul style="list-style-type: none"> - Basic Geometry - Proof by Contradiction
Learning Objectives	<ul style="list-style-type: none"> - Students can apply the proving skills learnt before - Students gain interest in the Isoperimetric Problem
Intended Learning Outcomes	<ul style="list-style-type: none"> - Students can apply the proving skills flexibly, choosing a correct proving method to solve the problems, making use of formula learnt or related prior knowledge, and writing down the negation of the statement when writing proof by contradiction - Students are committed to solving the problems, by trying to sketch figures to get ideas about the problems and asking teaching for hints
Learning & Teaching Strategies	Guided Discovery Activities, Group Discussion

Pre-lesson Task

Visit the websites and finish Pre-lesson Worksheet 5:

https://en.wikipedia.org/wiki/Bretschneider%27s_formula

Procedure

Learning Focus (Time)	Activity / Content	Learning & Teaching Strategies
Pre-lesson knowledge	<ol style="list-style-type: none"> 1. Students share their pre-lesson findings with reference to the Pre-lesson Worksheet 5. 2. Teacher explains some difficult parts in the proof of Bretschneider's Formula. 	Pre-lesson Worksheet 5
Problem-solving	Students work in groups to solve the problems. Teachers can provide hints when necessary. Students then present their solutions.	Lesson Worksheets 10 - 11
Summary	<ol style="list-style-type: none"> 1. Teacher summarizes the previous results and the relation to Problem (K) on the worksheet. 2. Teacher introduces the Isoperimetric Problem: <ul style="list-style-type: none"> - For a closed curve with fixed length, what kind of curve encloses the greatest area? - Students may already know that the answer is Circle. Teacher can tell about the importance of the problem. For example, some walled cities in the history were in circular. 	

Extended Learning Activity

Teacher can suggest students to move from Problem (K) to the investigation of Isoperimetric Problem using Unit H - Extension Materials.