Unit G

Suggested Answers and Guidelines

Suggested Answers and Guidelines for Unit G

Pre-lesson Worksheet 5

(1) For triangle, ‘all three sides have equal lengths’ implies ‘all three angles are equal’. Is ‘all sides have equal lengths’ implies ‘all angles are equal’ for polygon with more than three sides?

No. Counter-example: Square and Rhombus.

(2) Bretschneider’s Formula

a, b, c and d are the fixed lengths of the quadrilateral

and s is the semi-perimeter $\left(s = \dfrac{a+b+c+d}{2}\right)$

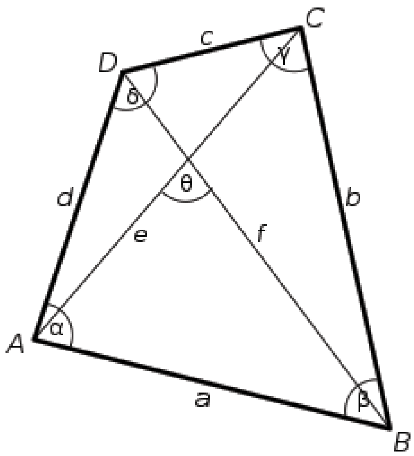
Scan the QR code and write down the Bretschneider’s Formula

Proof of Bretschneider’s Formula

$$Area = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\left(\frac{\alpha+\gamma}{2}\right)}$$

The complete proof is given in:
https://en.wikipedia.org/wiki/Bretschneider%27s_formula

Read the proof of the formula and write down four key formulas used in the proof.



1. <u>**Area Formula of Triangle**</u>

2. <u>**Cosine Formula**</u>

3. <u>**Compound Angle Formula**</u>

4. <u>**Double-Angle / Half-angle Formula**</u>

Suggested Answers and Guidelines

Lesson Worksheet 10

Problem (J)

Given four fixed lengths (satisfying quadrilateral inequality), prove that the area of the quadrilateral is maximum when it is a cyclic quadrilateral.

Hints/Guiding questions (Provide to students when necessary):

From the formula $Area = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\left(\frac{\alpha+\gamma}{2}\right)}$,

When will the area be maximum?

Solution to Problem (J)

From the formula $Area = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\left(\frac{\alpha+\gamma}{2}\right)}$

To have maximum area ,

$$\cos^2\left(\frac{\alpha+\gamma}{2}\right) = 0$$

$$\frac{\alpha+\gamma}{2} = 90°$$

$$\alpha+\gamma = 180°$$

Therefore, the opposite angles of the quadrilateral are supplementary.

By the result of Problem (F) , the quadrilateral is a cyclic quadrilateral.

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Lesson Worksheet 11

Prove that the area of regular polygon is larger than that of equilateral polygon under fixed perimeter.

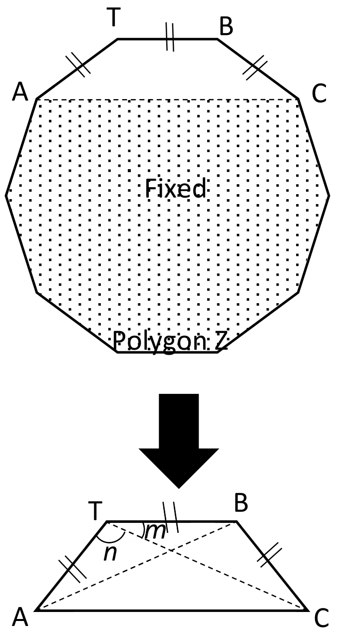
<u>Hints/Guiding questions (Provide to students when necessary):</u>

1. Try to use Proof by Contradiction
2. From the results and assumptions in Unit 6, we already know that the polygon maximizing the area exists and it should be convex and equilateral.
3. Try to sketch a figure following point 2.
4. Suppose the statement of point 2 is not true, what would 'NOT regular' mean?

   It is NOT regular while it is convex and equilateral. This means it is equilateral but not equiangular. This also implies that there is a pair of unequal adjacent angles.
5. Try to locate 4 consecutive points in an equilateral polygon, what can you observe?
   How can you maximize the area of the quadrilateral formed by these 4 points?
   By joining the two diagonals, what angle relations can be observed to be equal?

<u>Solution to Problem (K)</u>

Let Z be an equilateral polygon that maximizes the area for given perimeter P.
Assume Z is not equiangular, i.e. at least 1 pair of adjacent angles unequal
i.e $\angle ATB \neq \angle TBC$



AT, TB, BC and AC are fixed
Refer to the result of Problem(J),
$ATBC$ is the greatest area when it is a cyclic quadrilateral.

$\angle CTB = m$
$= \angle TCB$ (base $\angle$s, isos $\triangle$)
$= \angle BAT$ ($\angle$s in the same segment)
$= \angle ABT$ (base $\angle$s, isos $\triangle$)
So, $\angle BTC = \angle ABT = m$
$\angle ATC = \angle ABC = n$ ($\angle$s in the same segment)

So, $\angle ATB = \angle TBC = m + n$
This contradicts to the assumption that $\angle ATB \neq \angle TBC$

Polygon Z is a regular polygon that maximize the area under fixed perimeter