Unit E

Suggested Answers and Guidelines

Suggested Answers and Guidelines for Unit E

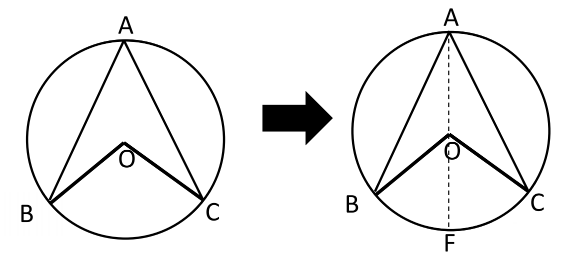
Pre-lesson Worksheet 3

<u>Property of Circle</u>

(1) Angle at centre twice angle at circumference

Given: O is centre of the circle and A, B and C lie on the circle.

Type 1



Prove $\angle BOC = 2\angle BAC$

Proof:

Extend AO to the circumference at F

Let $\angle BAF = x$ and $\angle CAO = y$

$OA = OB = OC$ (radii)

In $\triangle OAB$,

$\angle BAF = \angle ABO = x$ (base $\angle$s, isos $\triangle$)

$\angle BOF = 2x$ (ext. $\angle$ of $\triangle$)

In $\triangle OAC$,

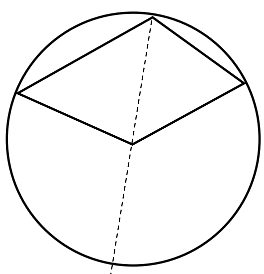
$\angle CAF = \angle ACO = y$ (base $\angle$s, isos $\triangle$)

$\angle COF = 2y$ (ext. $\angle$ of $\triangle$)

$\angle BOC = 2(x + y)$

$\quad\quad = 2\angle BAC$

Type 2



Prove $reflex\angle AOC = 2\angle ABC$.

Extend BO to the circumference at F

Let $\angle ABO = x$ and $\angle CBO = y$

$OA = OB = OC$ (radii)

In $\triangle OAB$,

$\angle BAO = \angle ABO = x$ (base $\angle$s, isos $\triangle$)

$\angle AOF = 2x$ (ext. $\angle$ of $\triangle$)

In $\triangle OAC$,

$\angle CBO = \angle BCO = y$ (base $\angle$s, isos $\triangle$)

$\angle COF = 2y$ (ext. $\angle$ of $\triangle$)

$reflex\angle AOC = 2(x + y)$

$\quad\quad = 2\angle ABC$

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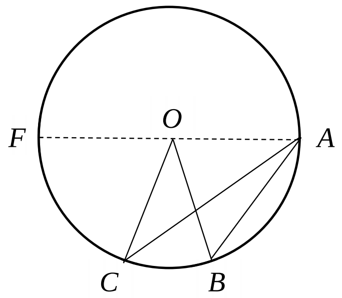
Type 3



Prove $\angle BOC = 2\angle BAC$.

Extend AO to the circumference at F

Let $\angle ABO = x$ and $\angle OCA = y$

$OA = OB = OC$ (radii)

In $\triangle OAB$,

$\angle BAO = \angle ABO = x$ (base $\angle$s, isos $\triangle$)

$\angle BOF = 2x$ (ext. $\angle$ of $\triangle$)

In $\triangle OAC$,

$\angle OCA = \angle OAC = y$ (base $\angle$s, isos $\triangle$)

$\angle COF = 2y$ (ext. $\angle$ of $\triangle$)

$\angle BOC = \angle BOF - \angle COF = 2(x - y)$

$= 2\angle BAC$

(2) Cyclic Quadrilateral

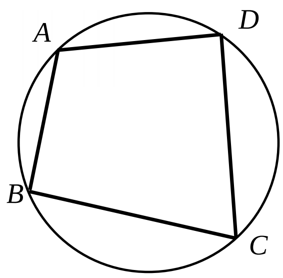
(a) What is Cyclic Quadrilateral?

A cyclic quadrilateral is a quadrilateral drawn inside a circle so that its vertices lie on the circumference of the circle.

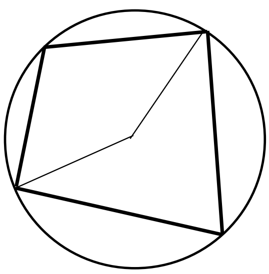
(b) If $ABCD$ is a cyclic quadrilateral, what result can we obtain?

$\angle A + \angle C = 180°$

$\angle B + \angle D = 180°$ (opp. $\angle$s, cyclic quad.)



(c) Prove the result in (b) by using "Angle at centre twice angle at circumference".



Prove: $\angle A + \angle C = 180°$.

$reflex\angle BOD = 2\angle A$ ($\angle$ at centre twice $\angle$ at circumference)

$\angle BOD = 2\angle C$ ($\angle$ at centre twice $\angle$ at circumference)

$reflex\angle BOD + \angle BOD = 360°$ ($\angle$s at a point)

$2\angle A + 2\angle C = 360°$

$\angle A + \angle C = 180°$

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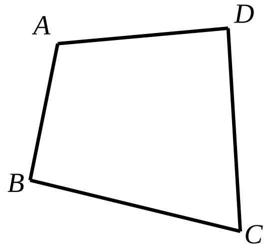
Lesson Worksheet 6

Problem (F)



If $\angle ABC + \angle ADC = 180°$, then *ABCD* is a cyclic quadrilateral.

Hints/Guiding questions (Provide to students when necessary):

(1) Is Converse of a true statement always true?

No.

(2) By your observation or do some trial construction, do you think the converse in Problem (F) is true?

Expect the students to answer 'Yes'.

(3) Can we draw a circle passing through *A*, *B* and *C*?

From previous lesson, any three non-collinear points are always concyclic.

There is a unique circle passing through A, B and C

(4) After drawing a circle passing through *A*, *B* and *C* . what happen to *D* if *ABCD* is a cyclic quadrilateral? What happen to *D* if *ABCD* is not a cyclic quadrilateral?

If *ABCD* is a cyclic quadrilateral, *D* lies on the same circle.

If *ABCD* is not a cyclic quadrilateral, *D* lies either inside or outside the same circle.

(5) Rewrite the negation (The result when the statement is wrong) of the statement.

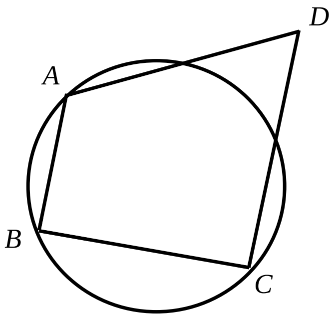
'If $\angle ABC + \angle ADC = 180°$, then *ABCD* is a cyclic quadrilateral.'

" $\angle ABC + \angle ADC = 180°$ but *ABCD* is not a cyclic quadrilateral"

(6) Now we assume the result to be wrong (i.e. $\angle ABC + \angle ADC = 180°$ but *ABCD* is not a cyclic quad.). What do we need to do in order to prove that the original statement is true?

To show that it is impossible to assume it to be wrong. We should see some contradictions.

(7) Suppose *D* is outside the circle, draw a figure.



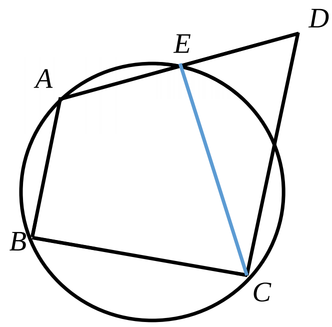
(8) How to make use of the condition $\angle ABC + \angle ADC = 180°$?

How to make use of the circle and the property of circle?

Suggested Answers and Guidelines

Solution to Problem (F)

Target: Prove that 'if $\angle ABC + \angle ADC = 180°$, then $ABCD$ is cyclic quadrilateral.'



Assume that the statement is false,

i.e. $\angle ABC + \angle ADC = 180°$ but $ABCD$ is not a cyclic quadrilateral.

Construct the circumcircle of ABC

By assumption, D lies either inside or outside the circle.

Consider the case D lies outside the circle.

Then AD cuts the circle at E

Join CE as shown, $ABCE$ is a cyclic quadrilateral by construction.

i.e. $\angle ABC + \angle AEC = 180°$ (opp. $\angle$s, cyclic quad.)

From the given condition, $\angle ABC + \angle ADC = 180°$

Compare the two equations , we have $\angle AEC = \angle ADC$

Hence, $EC \,//\, DC$ (corr. $\angle$s equal),

which contradicts to the fact that EC and DC intersect at C .

So, the assumption cannot happen.

Therefore, if $\angle ABC + \angle ADC = 180°$, then $ABCD$ is cyclic quadrilateral.

The above proof is an example of 'Proof by Contradiction', which is the common approach for mathematicians.

Unit E

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Lesson Worksheet 7

Problem (G)

Given a circle with centre O, and A and B are two points on the circle. If P is a point such that $\angle AOB = 2\angle APB$, where O and P lying on the same side of chord AB, prove that P also lies on the circle.

<u>Hints/Guiding questions (Provide to students when necessary):</u>

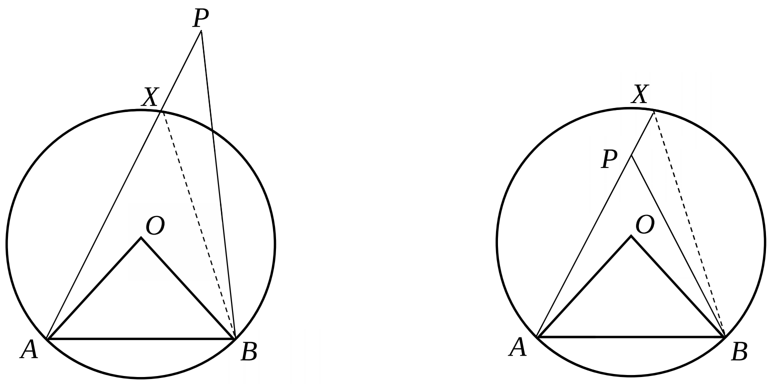
Try to use 'Proof by Contradiction'.

1. Draw a figure with the triangle ABP which does not satisfy the statement.
2. Let the point join line AP and the circumference be X.
3. Make use of property of circle and assumptions to look for contradiction.

<u>Solution to Problem (G)</u>

Assume the statement is wrong, i.e. P is a point such that $\angle AOB = 2\angle APB$ but P does not lie on the circle.

P lies either outside or inside the circle as shown in the following figure.



Let X be the intersection of line AP and the circumference.

By assumption, $\angle AOB = 2\angle APB$.

In the circle, $\angle AOB = 2\angle AXB$ ($\angle$ at centre twice $\angle$ at circumference)

Compare the two equations, $\angle APB = \angle AXB$

Hence, $BX \parallel BP$ (corr. $\angle$s equal)

This contradicts to the fact that BX and BP meet at B.

Therefore, the statement is proved.