Unit D

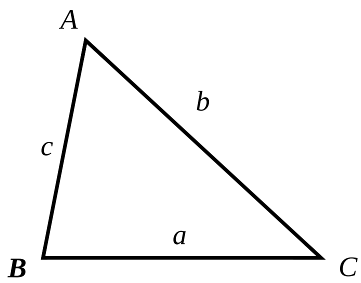
Suggested Answers and Guidelines

Suggested Answers and Guidelines for Unit D

Pre-lesson Worksheet 2

<u>Triangle Inequality</u>

Proof of Triangle inequality



Write down the triangle inequality.

$a+b>c$

$b+c>a$

$a+c>b$

Draw triangle ABC and the line perpendicular to BC passing through vertex A.

Now prove that BA + AC > BC

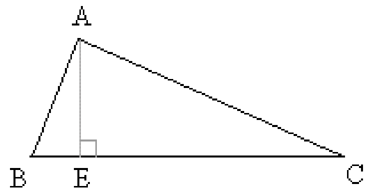
BE is the shortest distance from vertex B to AE , This means BA > BE

CE is the shortest distance from C to AE. This means AC > CE

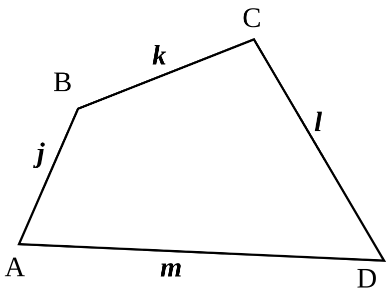
BA > BE and AC > CE

Add the left side and add the right of the inequalities. This gives:

BA + AC > BE + CE = BC



Proof of Quadrilateral inequality



(A) Guess the quadrilateral inequality.

$j+k+m>l$

$m+l+k>j$

$l+k+j>m$

(B) Verify it by using the triangle inequality.

Join AC as shown.

By Triangle Inequality

$j+k>n$ and $n+m>l$

$j+k+m>n+m$

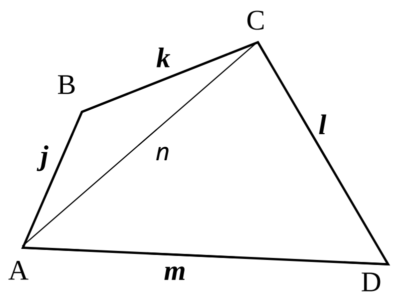
$j+k+m>l$

Similarly

$m+l>n$ and $n+k>j$

$m+l+k>n+k$

$m+l+k>j$



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Lesson Worksheet 4

Problem (D1)

If you have three given lengths (satisfying the Triangle Inequality) , how many different triangles can you make from these lengths?

Solution to Problem (D1)

A unique triangle can be formed due to the condition for congruent triangles (S.S.S.) .
Or we can use cosine formula to find each angle.

Problem (D2)

If you have four given lengths (satisfying the Quadrilateral Inequality), is there only ONE quadrilateral that can be formed? Explain your answer.

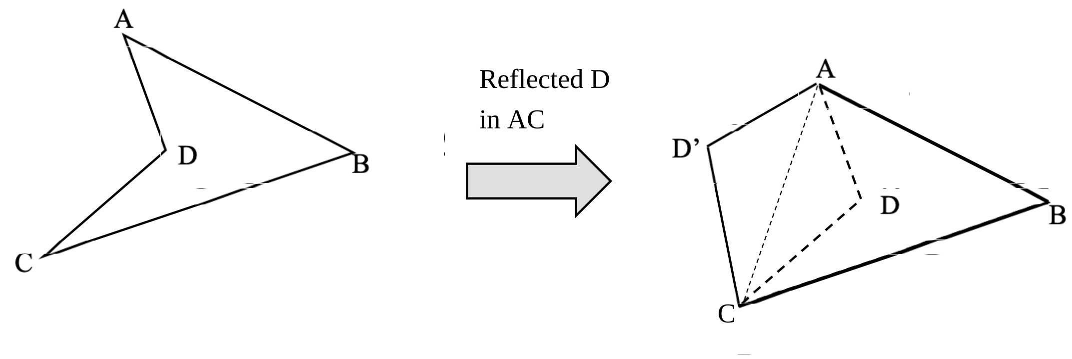
Guideline: If yes, give reasons logically. If not, just give one Counter-example.

Solution to Problem (D2)

Some counter-examples can be suggested to students:

(a) Concave

Convex, keeping all 4 lengths equal



(b) Rhombus and Square
(c) Parallelogram and Rectangle

Suggested Answers and Guidelines

Lesson Worksheet 5

Problem (E1)

Prove any three non-collinear points always concyclic.

Guideline 1:

Let A , B and C be the three non-collinear points.

To prove the statement, we need to find out a way to construct a circle passing A , B and C

To construct the circle, we need to locate the centre.

Guideline 2:

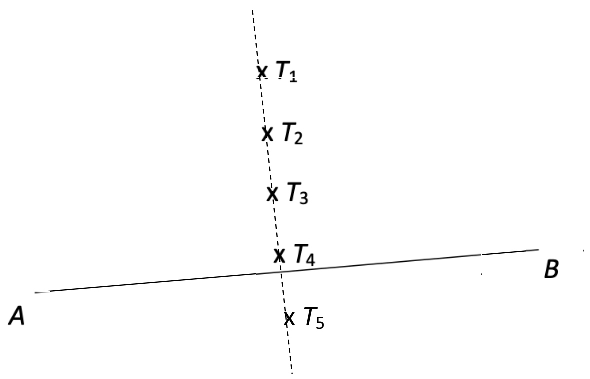
Suppose X is the centre of the circle we want, X should satisfy the property $XA = XB = XC$.

We can begin with the following questions:

(a) A and B are fixed point, is it possible to find a point T such that $AT = BT$?

(b) Other than point T, any other point(s) also equidistant from A and B ?

(c) Joining all the points to become a straight line, what's the relationship between the straight line and AB ?



The above questions guide students to conclude that X lies on the perpendicular bisector of AB .

Guideline 3:

There are perpendicular bisectors of AB, BC and AC respectively, do they intersect at a point?

Teacher may ask them to roughly draw (or construct by compass and straight edge if they have leant before) the perpendicular bisectors to get intuitive ideas before jumping to the proof that the perpendicular bisectors always concurrent.

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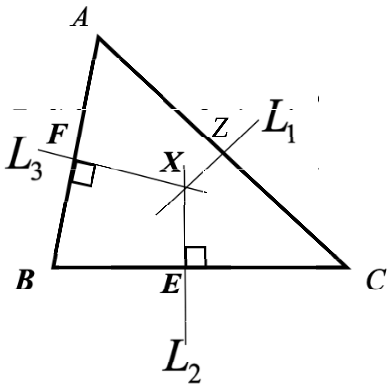
Solution to Problem (E1)
[Proof that three perpendicular bisectors are concurrent]

Let L_2 and L_3 be the perpendicular bisectors of BC and AB respectively

Denote the point of intersection of L_2 and L_3 as X



$\Delta XBE \cong \Delta XCE$ (S.A.S)

$\Delta XBF \cong \Delta XAF$ (S.A.S)

$\therefore XA = XB = XC$

Let Z be the mid-point of AC

$\Delta XAZ \cong \Delta XCZ$ (S.S.S)

$\angle AZX = \angle CZX = 90°$

L_1 is the perpendicular bisector of AC .

Therefore,

ΔABC lies on the circle with centre X and radius AX

GeoGebra Reference: https://www.geogebra.org/m/ebpxRqk8

Problem (E2)

Is it true that any four points (not three of them are collinear) are always concyclic?

Solution to Problem (E2)

A counter-example can be constructed by first constructing a circle passing through three points, and the choose a point outside / inside the circle to be the 4^{th} point.