Unit H

Extension Materials

<u>Unit H Extension : Isoperimetric Problem</u>

In the programme, we come to the conclusion:
The area of regular polygon is the largest under fixed perimeter.

Following the steps below, we can move on to show that circle encloses the largest area among all closed curve with fixed length. See the following websites to continue your investigations.

(1) A regular polygon with *i* sides has greater area than one with *j* sides under a fixed perimeter if and only if $i > j$.

Formula of area of regular polygon

Changing area of regular polygon by using GeoGebra

(2) The second part involves the concepts of radian measures and limits, which will be introduced in M1/M2 syllabus.

Evaluating $\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$

When $n \to \infty$,
Area of regular polygon

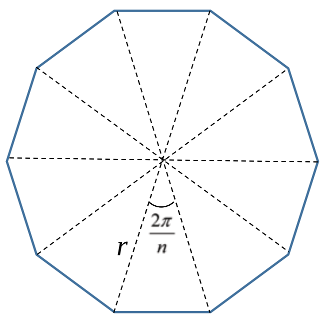
$$= \lim_{n\to\infty}\left[n \cdot \frac{1}{2} r^2 \sin\left(\frac{2\pi}{n}\right) \right]$$

$$= \frac{r^2}{2} \lim_{n\to\infty} \frac{\sin\left(\frac{2\pi}{n}\right)}{\frac{2\pi}{n} \cdot \frac{1}{2\pi}}$$

$$= 2\pi \cdot \frac{r^2}{2} \lim_{n\to\infty} \frac{\sin\left(\frac{2\pi}{n}\right)}{\frac{2\pi}{n}}]$$

$$= \pi r^2$$

n-sided regular polygon



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Therefore, **CIRCLE** encloses maximum area under fixed perimeter.

(3) The reasoning about Isoperimetric Problem is almost complete, except that in Unit 6, we have the assumption:

Under fixed perimeter and number of sides, there is a polygon with the greatest area.

The following document provides a detailed discussion about the problem, which also proves the above assumption using advanced mathematics.

From the Triangle Inequality to
the Isoperimetric Inequality