Unit C

Suggested Answers and Guidelines

Suggested Answers and Guidelines for Unit C

Pre-lesson Worksheet 1

1. $y = \sin x$, $0° \le x \le 360°$

   a. What is the range of $\sin x$?
   $-1 \le \sin x \le 1$

   b. What is the maximum value of $\sin x$ and the corresponding value of x ?
   $\sin x = 1, x = 90°$

   c. What is the minimum value of $\sin x$ and the corresponding value of x ?
   $\sin x = -1, x = 270°$

2. $y = \cos x$, $0° \le x \le 360°$

   a. What is the range of $\cos x$?
   $-1 \le \cos x \le 1$

   b. What is the maximum value of $\cos x$ and the corresponding value of x ?
   $\cos x = 1, x = 0°$ or $360°$

   c. What is the minimum value of $\cos x$ and the corresponding value of x ?
   $\cos x = -1, x = 180°$

3. Are there any characteristics in these two graphs?
   (Any reasonable answers)

4. Compound angle formulae

   a. Write down the formulae of $\sin(A+B)$ and $\cos(A+B)$.

   $\sin(A+B) = \sin A\cos B + \cos A\sin B$

   $\cos(A+B) = \cos A\cos B - \sin A\sin B$

   b. Using the above result, find the formulae of $\sin 2A$ and $\cos 2A$.

   $\sin 2A = 2\sin A\cos B$
   $\cos 2A = \cos^2 A - \sin^2 B$

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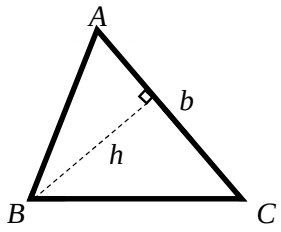
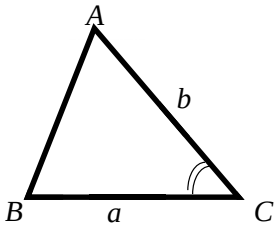
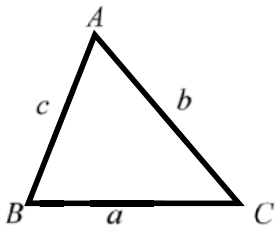
Lesson Worksheet 1

Hints/Guiding questions (Provide to students when necessary)

<table>
<tr><td colspan="3">Area of triangle can be obtained by</td></tr>
<tr>
<td>(1) $A = \frac{bh}{2}$
 
</td>
<td>(2) $A = \frac{1}{2}ab\sin C$
 
</td>
<td>(3) $A = \sqrt{s(s-a)(s-b)(s-c)}$
where $s = \frac{a+b+c}{2}$
 
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</tr>
</table>

Which formulae is suitable for this case? Why?

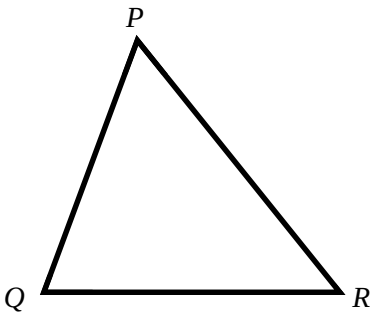
(2) $A = \frac{1}{2}ab\sin C$.

We can set the two fixed lengths as a and b, leaving the angle C to be the only variable.

What is the maximum value of $\sin x$?

Maximum value = 1

Solution to Problem (A)



If the lengths of QR and PQ are fixed numbers, when does the triangle attain the greatest area? Explain your answer.

$0° < \angle PQR < 180°$

$0 < \sin \angle PQR \le 1$

$\sin \angle PQR = 1$ when $\angle PQR = 90°$

Area $= \frac{1}{2}(PQ)(QR)\sin \angle PQR$

Area is maximum when $\sin \angle PQR = 1$.

Area is maximum when $\angle PQR = 90°$.

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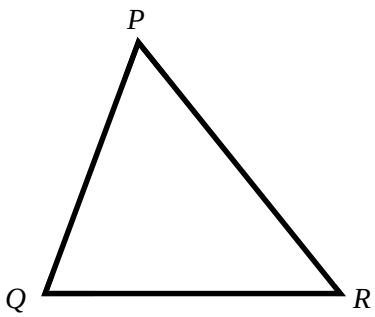
Lesson Worksheet 2

Hints/Guiding questions (Provide to students when necessary)

Hints (1) Use Heron's Formula to find the area of triangle.
Hints (2) (1) Replace $12-a$ by t. (2) After replacement, which one is variable?
Hints (3) Use completing the square.

Solution to Problem (B)



It is given that the perimeter of a triangle PQR is 24 cm. If the length of QR is a cm, where a is constant and P is a moving point, when does the triangle attain the greatest area? Explain your answer.

Assume $PQ + PR = 2t$, where t is a constant

$PQ = t + x$ *and* $PR = t - x$, where x is a variable

$$\text{Area} = \sqrt{12(12-a)[12-(t-x)][12-(t+x)]}$$

$$= \sqrt{12(12-a)[(12-t)+x][(12-t)-x]}$$

$$= \sqrt{12(12-a)[(12-t)^2 - x^2]}$$

$\because$ 12 , $(12-a)$ and t are constants

$\therefore$ $(12-t)^2 - x^2$ is maximum when $x = 0$

When $x = 0$, $PQ = PR$

When PQR is an isosceles triangle with $PQ = PR$, it attains the greatest area.

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Lesson Worksheet 3

Solution to Problem (C)

It is given that the perimeter of rectangle *ABCD* is 24 cm. When does the rectangle attain the greatest area?

Let $AB = x$, where x is a variable

$$\begin{aligned} Area &= x(\frac{24}{2} - x) \\ &= x(12 - x) \\ &= -x^2 + 12x \\ &= -(x-6)^2 + 36 \end{aligned}$$

Area is maximum when $x = 6$, which means when it is a square.