Unit F

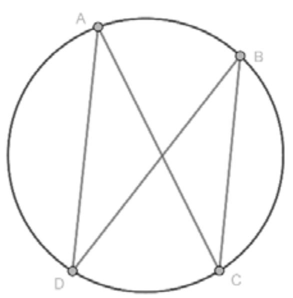
Suggested Answers and Guidelines

Suggested Answers and Guidelines for Unit F

Pre-lesson Worksheet 4

(1) Angles in the same segment

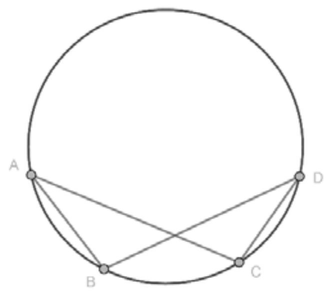
Type 1



Prove $\angle DAC = \angle DBC$.

Type 2

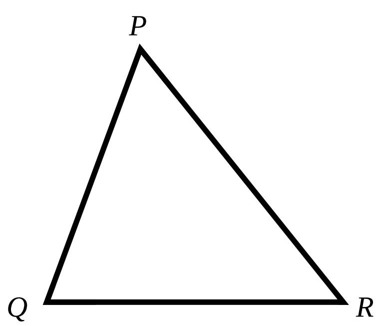
Prove $\angle ABD = \angle ACD$.



<u>Hints: Add the centre of the circle and apply angles at centre twice angle at circumference.</u>

The following problem will be useful in the coming lesson. Try it again.

Review: <u>Problem (B)</u>



It is given that the perimeter of a triangle PQR is 24 cm . If the length of QR is a cm, where a is constant and P is a moving point, when does the triangle attain the greatest area? Explain your answer.

<u>Refer to Unit 3 Problem (B)</u>

Suggested Answers and Guidelines

Lesson Worksheet 8

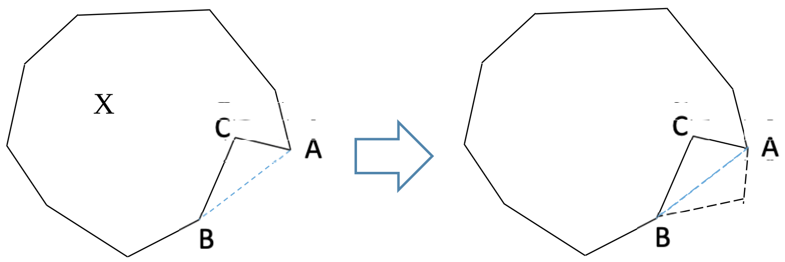
Solution to Problem (H)

Given a concave polygon, describe how to construct a convex polygon with the same perimeter and a larger area than the concave polygon.

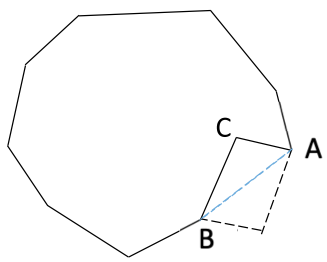
Assume X is a polygon which is not convex

Reflect *C* along *AB*. This will keep the length of *AC* and *BC* unchanged.

Keep doing this for any concave points until the polygon become convex.



Or : Construct a parallelogram with diagonal *AB* as shown.



In Mathematics inquiry, suggesting a way to construct the desired results is an important skill. To be more precise, teacher can ask students to suggest how the above construction works using straight edge and compasses.

Unit F

Suggested Answers and Guidelines

Lesson Worksheet 9

Assumption:

Under fixed perimeter and number of sides, there is a polygon with the greatest area.

Teacher may explain the importance of this assumption after finishing problem (I).
This assumption can be proved by more advanced Mathematics.

Problem (I):

Prove that the area of equilateral polygon is larger than that of irregular polygon under fixed perimeter.

<u>Hints/Guiding questions (Provide to students when necessary):</u>

We can use Proof by Contradiction :

If the polygon with the greatest area is NOT equilateral, what happens?

Equilateral means all sides have equal lengths.

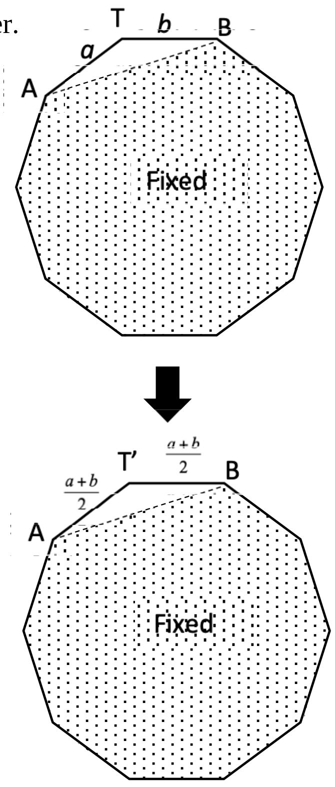
Not equilateral means at least one pair of adjacent sides have unequal lengths.

<u>Solution to Problem (I)</u>

By the assumption, there is a polygon with the greatest area.
From the result of Problem (H), the polygon should be convex.

Let Y be a convex polygon that maximizes the area for given perimeter.
Assume Y is not equilateral polygon,
i.e. at least 1 pair of adjacent sides unequal ($AT \neq BT$)
i.e. $a \neq b$ but $a+b$ is fixed

Refer to the Problem (B) in Unit 3,
When T moves to T' with $AT' = BT'$,
($\Delta AT'B$ is an isosceles triangle)
the area of $\Delta AT'B$ is greater than that of ΔATB.



The perimeter of the new polygon is equal to that of polygon Y
and the area of the new polygon is greater than that of polygon Y.

This contradicts the assumption that Y is the polygon that maximizes the area for given perimeter.

Therefore, the area of equilateral polygon is larger than that of irregular polygon under fixed perimeter.